Coalition Formation in Real Time with an Application to Bargaining over Customs Union

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CEPET Workshop Udine 2015

Outline

A Customs Union Formation Problem

Following Aghion/Atras/Helpman (2006)

- Customs union formation game as example of game with externality.
- Three agents may form a customs union {A,B,C}.
- The default outcome with no union is $(1, 1, 1)$
- The union (=grand coalition) has a total value of 6 which can be distributed (such as $2, 2, 2$).
- Assume a core customs union {A,B} forms: Due to the externality we obtain $(2, 1, 0)$.
- Will efficient {A,B,C} form or will we endure one period of inefficiency from which an improvement to $(5 - 2\epsilon, 1 + \epsilon, \epsilon)$ in period 2 is possible?

Customs Union Formation

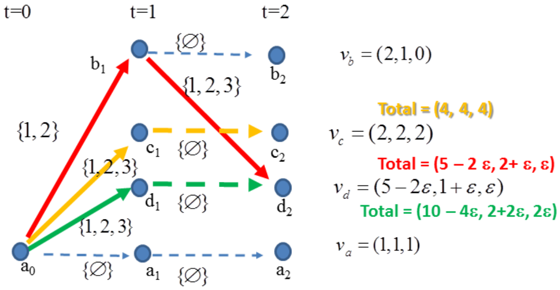


Figure 1: Customs Union Formation

An Inefficient Design?!

- There might be examples where the inefficient red path is realized.
- Yet applying the "spirit" of subgame perfection suggests
  - if C is offered the green path
  - and she is sophisticated
  - she should accept

- One way: Model institutions in detail and use non cooperative bargaining theory
  - Gomez/Jehiel (2005) model spot contract proposals which can only be accepted or rejected and obtain red
  - Aghion/Atras/Helpman suggest a "closed-loop" bargaining framework and also obtain red
  - Alibekos/Madumarov/Pech (2015) suggest a "open-loop bargaining protocol" which supports green
  - As do Gomez/Jehiel (2005) with contingent contracts
  - Acemoglu/Egorov/Sonin: Always a protocol to get efficiency
- The general idea in the coalition formation literature is to impose on feasible paths that the endpoint indirectly dominates the starting point (see e.g. Konishi/Ray, 2003)
- But where for "institution-free bargaining" is the equivalent of subgame perfection?

The Stable Set of Paths

- We define a stable set on paths in real time (unidirectional)
- which makes the same predictions as subgame perfection when this is applicable
- and is capable of incorporating different behavioral assumptions in the path structure (myopia, farsightedness)

The Stable Set of Paths

- Previous approaches:
  - Xue (1998) defines *OSSB* and *CSSB* for graphs but assigns empty set to nodes when there is more than one path which may be taken
  - Tadelis (1996) defines *OSSB* for repeated games and obtains refinement of subgame perfection (but cannot support a second solution)

Time Paths

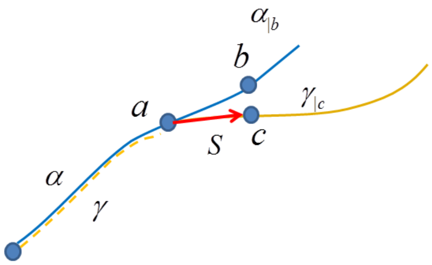
- Let Z_t be the set of nodes associated with time period t
- Inducement correspondence: Coalition S can move from a to b ($a \xrightarrow{S} b$)
- This implies that if $a \in Z_t$ then $b \in Z_t$ or $b \in Z_{t+1}$
- If some node c can be reached from a then there exists a path α with $a, c \in \alpha$.

Domination via $\triangleleft$

Definition 1

Assume that node b is a direct successor of a on path α. A (truncated) path $\gamma_{|a}$ dominates path α via $\triangleleft$, i.e. $\alpha \triangleleft \gamma_{|a}$, if there is a node $a \in \alpha \cap \gamma$ and $a \stackrel{S}{\longrightarrow} c$, $c \in \gamma$, $c \notin \alpha$ and for the truncated paths $\alpha_{|b}$ and $\gamma_{|c}$: $\gamma_{|c} \succsim_S \alpha_{|b}$.

γ dominates α

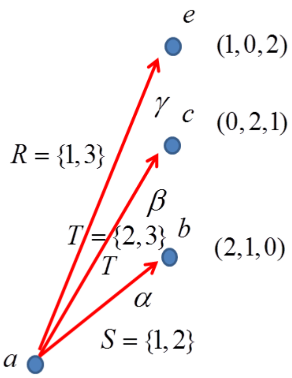


Cyclicality at node *a*

Definition 2

$\triangleleft$ is cyclical at node a if there are $K > 2$ paths $\alpha_k \in X(a)$ such that (1) there is a sequence $\alpha_k \triangleleft \alpha_{k+1}$, $k = 1, ..., K - 1$, with $\alpha_K \triangleleft \alpha_1$ and (2) there is no path α_j such that $\alpha_j \succsim_{S_j} \alpha_k$ for all $k = 1, ..., K$.

Cyclicality at node *a*

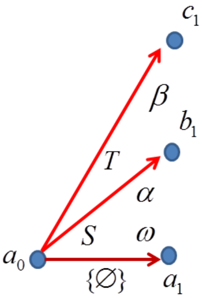


Default Path

Definition 3

In each node *a* there is one path which originates and for which the empty coalition is effective. We call this path default path and use $\omega(a)$ as a label for this path.

Default Path



Domination by the Default Path

Definition 4

Let α be a path originating in node *a* with effective coalition S_α and $\omega(a)$ the corresponding default path. If $\alpha \not\succ_{S_\alpha} \omega$ then $\alpha \triangleleft \omega$.

- Quite natural: If one agent objects he implicitly invokes the default path
- This could give raise to cycles involving the default path
- If someone likes better the idea that ω dominates α or β when it dominates both.
- This kind of strengthening does not matter for the solution

Stable Set

- Define the set of admissible paths Π

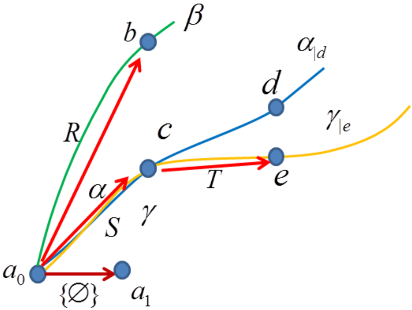
Definition 5

A set $V \subset \Pi$ is stable if it is internally stable and externally stable. It is internally stable if $\alpha \in V$ implies that there is no $\beta \in V$ such that $\alpha \triangleleft \beta$. And it is externally stable if for all $\gamma \notin V$ there is $\alpha \in V$ such that $\gamma \triangleleft \alpha$.

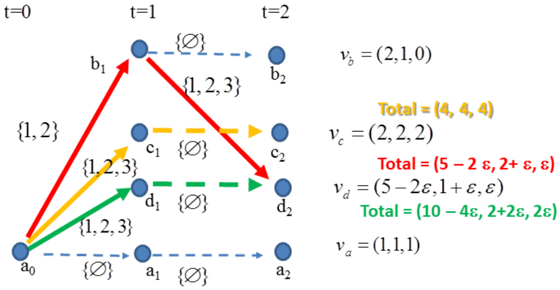
Proposition 6

Assume that at no node the dominance relation $\triangleleft$ *is cyclical. Then a stable set for* $(\Pi, \triangleleft)$ *exists.*

Stable Set: Existence



Stable Set of the Customs Union Game



Stable Set of the Customs Union Game

- *Green* and *Yellow* do not dominate each other
- *Red* is dominated by *Green*
- The default path is dominated by yellow
- So $V = \{Green, Yellow\}$

The Relationship between Stable Set and Subgame Perfection

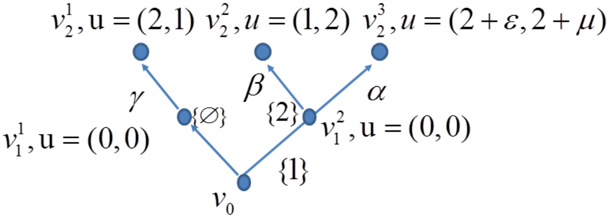
Proposition 7

Assume in each node only one agent moves and all nodes other than the terminal nodes give a pay off of zero. Moreover, assume that paths are finite. If all agents are sophisticated farsighted, the sets of stable sets on Π (weakly) includes the set of subgame perfect strategies of the corresponding extensive form game.

Corollary 8

For a strict preference order, this coincides with Kuhn's theorem

Extensive Form Game

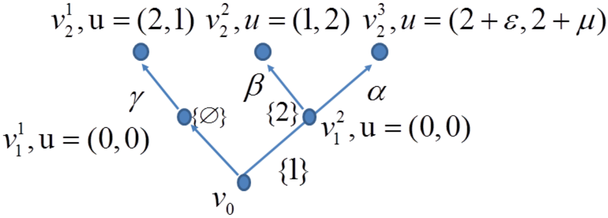


Extensive Form Game

$$\varepsilon > 0, \mu > 0 : SGPE = (\alpha)$$

$$\varepsilon > 0, \mu = 0 : \mathrm{SGPE}_1 = (\alpha), \mathrm{SGPE}_2 = (\gamma, \underline{\beta})$$

$$\varepsilon = 0, \mu = 0 : \mathrm{SGPE} = (\gamma)$$



Extensive Form Game

$$\varepsilon > 0, \mu > 0 : \mathrm{V} = \{\alpha\}$$

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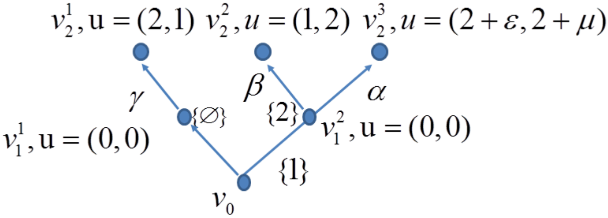


Refining the Stable Set

Definition 9

A path $\alpha \in \Xi(a)$ if in position a it is undominated via $\lhd\lhd$. A path α starting position in a with direct successor b is dominated by path γ, i.e. $\alpha \lhd \lhd \gamma$, if $a \in \alpha \cap \gamma$ and $a \xrightarrow{S} c$, $c \in \gamma$, $c \notin \alpha$ and for all paths $\delta_k \in \Xi(c)$: $\delta_k \succsim_S \beta_i \; \forall \beta_i \in \Xi(b)$ with at least one strict inequality.

Refining the Stable Set

Definition 10

A set $W \in \Pi$ is strictly stable if it is internally strictly stable and externally strictly stable. It is internally strictly stable if $\alpha \in W$ implies that there is no $\beta \in W$ such that $\alpha \lhd\lhd \beta$. And it is externally strictly stable if for all $\gamma \notin W$ there is $\alpha \in W$ such that $\gamma \lhd\lhd \alpha$.

Collecting All Paths Corresponding to Some SGPE

Lemma 11

Assume that the game tree is finite and that at each node only one agent moves and that each move is observed by all agents. Then at each node a path is in W if and only if it is played as part of a subgame perfect equilibrium. Moreover, a path is stable if and only if it coincides with the equilibrium path of some subgame perfect equilibrium.

Obtaining Subgame Perfection

Theorem 12

Assume that each edge of a game tree corresponds to a section of a path. Moreover, assume that the game tree is finite and that at each node only one agent moves and that each move is observed by all agents. Consider the (possibly non unique) stable set of Π*. Each set* $V \cap W$ *corresponds to one SGPE of a game on the graph* Π*.*

Summary

- The stable set defined on paths allows to incorporate credible threats into a coalition formation process in real time.
- It offers a flexible tool to deal with institutional details and different degrees of rationality
- The solution generally exists unless the dominance relation is cyclical at some node
- It generalizes subgame perfection

Appendix

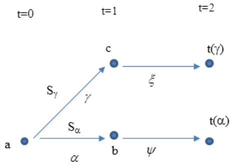
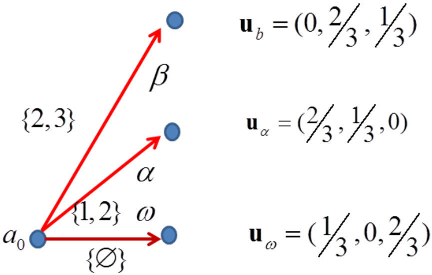


Figure 6: Illustration of the difference between definition 1 and the standard definition of domination between paths

Appendix


$\mathbf{u}_b = (0, 2/3, 1/3)$
$\mathbf{u}_\alpha = (2/3, 1/3, 0)$
$\mathbf{u}_\omega = (1/3, 0, 2/3)$
β
$\{2,3\}$
α
$\{1,2\}$
ω
a_0
$\{\varnothing\}$