Intra-party decision making, party formation, and moderation in multiparty systems: Examples

Gerald Pech*

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1 Details of example 1 in the main text

Example 1 *Consider a game with* $N = \{1, 2, 3, 4, 5\}$ *and* $u_i(x) = -|x - \widehat{x}^i|$, *with the distribution of bliss points* $\widehat{x}^1 = 0.1$, $\widehat{x}^2 = 0.2$, $\widehat{x}^3 = 0.5$, $\widehat{x}^4 = 0.55$, $\widehat{x}^5 = 1$ *and* $x^{sq} = 1$. *Agents are pessimistic.*

Partition $\pi_1 = \{\{1\}, \{2, 3, 4\}, \{5\}\}$ results in $M = \{2, 3, 4\}$ and gives the pay off vector $(-.3167, -.2167, -.1167, -.1333, -.5833)$, partition $\pi_2 = \{\{1, 2\}, \{3, 4\}, \{5\}\}$ results in $M = \{\{1, 2\}, \{3, 4\}\}$ and gives $(-.2375, -.1875, -.1875, -.2125, -.6625)$, partition $\pi_3 = \{\{1, 2\}, \{3, 4, 5\}\}$ results in $M = \{3, 4, 5\}$ and gives $(-.1667, -.1333, -.2333, -.2833, -.7333)$ as does partition $\pi_4 = \{\{1\}, \{2\}, \{3, 4, 5\}\}$ and partition $\pi_5 = \{\{1\}, \{2\}, \{3, 4\}, \{5\}\}$ gives $(-.3167, -.2167, -.2333, -.2833, -.7333)$. These partitions dominate every other partition in Π, i.e. there is a deviation by a coalition which results in π_1, π_2, π_3, π_4 or π_5. Yet none of these partitions is stable: Coalition $\{1, 2\}$ wants to deviate from π_1 and π_5 and induces π_2, coalition $\{3, 4, 5\}$ wants to deviate from π_2 and induces π_3. Coalition $\{2, 3, 4\}$ wants to deviate from π_3 and π_4, inducing π_1.

*KIMEP, e-mail: gp@geraldpech.net

2 Details of example 2 in the main text

Example two of the main text demonstrates that the following remark is true:

Remark 1 *In a stable party structure, $x \in Z_M$ and $x \prec_m x^{sq}$ is possible.*

Example 2 *Consider a game with $x^{sq} = 0.6$, $n = 11$ and $u_i(x) = -|x - \widehat{x}^i|$ with the distribution of bliss points $\widehat{x}^1 = 0.35$, $\widehat{x}^2 = 0.35 + \varepsilon$, $\widehat{x}^3 = 0.4$, $\widehat{x}^4 = 0.454$, $\widehat{x}^5 = 0.499$, $\widehat{x}^6 = \widehat{m} = 0.5$, $\widehat{x}^7 = 0.5755$, $\widehat{x}^8 = 0.5755 + \varepsilon$ and $\widehat{x}^i = 0.6 + (i-9)\varepsilon$ for $i = 9, 10, 11$, $\varepsilon \to 0$. Agents are pessimistic.*

We want to show stability of partition $\pi^* = \{\{1\}, S, \{8\}, \{9\}, ..\}$ with $S = \{2, 3, 4, 5, 6, 7\}$. $G(\pi^*) = \{S\}$ is a singleton and in S, agent 5 is pivotal. If agent 2 proposes 0.398, agents 3, 4 and 5 support the proposal. Before showing that stability of π^* we show stability of $\pi_1 = \{\{1, 2, 3, 4, 5\}, \{6, 7\}, \{8\}, \{9\}, \{10\}, \{11\}\}$ and $\pi_6 = \{\{1, 2\}, \{3, 4, 5, 6, 7, 8\}, \{9\}, \{10\}, \{11\}\}$. The indices refer to the table.

Stability of π_1: π^* and π_6 are preferred by all agents $i \leq 6$. Yet deviations from π_1 to intermediate partitions such as $\pi = \pi_{12}$ or π_{13} are deterred because $\pi_1 \in \Xi(\pi)$ via π_{14} and π_2.

Stability of π_6: 1, 2, 3 and 4 prefer π^*. $\{1, 2, 3\}$ may deviate to $\pi = \pi_{15}$ or $\{1, 2, 3, 4\}$ to $\pi = \pi_{16}$, yet for each π, $\pi_6 \in \Xi(\pi)$ via a deviation of $\{5, 6\}$ to π_{17} (π_{18}).

Now we can show stablitity of π^*.

First, consider deviations by $i \geq m$. π_1 is stable (which rules out stability of partitions π_2 and π_3) and preferred by $i \geq 7$. However, 6 cannot be forced into π_1: 7 may bring about π_4, but $\pi^* \in \Xi(\pi_4)$ (via π_5 and π_2).

$\{5, 6, 7, 8\}$ would prefer stable π_6 over π^*. Deviations of $\{5, 6, 7\}$ to π_8 or of $\{5, 6\}$ to π_9 or of $\{6\}$ to $\pi'_9 = \{\{1\}, \{2, 3, 4, 5\}, \{6\}, ...\}$ - resulting in the same pay off as π_9 - are deterred because for each π, $\pi^* \in \Xi(\pi)$. A deviation of $\{5, 6, 7, 8\}$ to π_7 is deterred because $\pi^* \in \Xi(\pi_7)$ via π_9.

A deviation of $\{4, 5, 6, 7\}$ to π_{10} by is deterred because $\pi^* \in \Xi(\pi_{10})$. Inclusion in government of agents further on the right than 8 is objected even by 5 and 6.

Furthermore, $\{1, 2, 3\}$ or $\{1, 2\}$ do not want to deviate because partitions which could be reached such as π_{11} are unattractive for the deviators.

3 Tables

Partition	government	U^1	U2	U^2	U^3	U^4	U^5	Deviation
π_1=1|234|5	234*	-0.31667	-0.21667	-0.21667	-0.11667	-0.13333	-0.583333	π_2=12|34|5
π_2=12|34|5	12,34*	-0.2375	-0.1875	-0.1875	-0.1875	-0.2125	-0.6625	π_3=12|345
	34,5	-0.9	-0.8	-0.8	-0.5	-0.45	0	
	12,5	-0.9	-0.8	-0.8	-0.5	-0.45	0	
π_3=12|345	345*	-0.58333	-0.48333	-0.48333	-0.18333	-0.16667	-0.316667	π_1=1|234|5
π_4=1|2|345	345*	-0.58333	-0.48333	-0.48333	-0.18333	-0.16667	-0.316667	π_1=1|234|5
	1,345	-0.4375	-0.3875	-0.3875	-0.2375	-0.2375	-0.4625	
	2,345	-0.4625	-0.3625	-0.3625	-0.2125	-0.2125	-0.4375	
π_5=1|2|34|5	2,34*	-0.31667	-0.21667	-0.21667	-0.11667	-0.13333	-0.583333	π_2=12|34|5
	1,34	-0.28333	-0.25	-0.25	-0.15	-0.16667	-0.616667	
	34,5	-0.9	-0.8	-0.8	-0.5	-0.45	0	
	1,2,5	-0.9	-0.8	-0.8	-0.5	-0.45	0	
	1,2,34	-0.2375	-0.1875	-0.1875	-0.1875	-0.2125	-0.6625	
π=1|23|4|5	min	-0.31667	-0.21667	-0.21667	-0.23333	-0.28333	-0.733333	π_5=1|2|34|5
	1,23 (M1)	-0.16667	-0.13333	-0.13333	-0.23333	-0.28333	-0.733333	
	23,4 (M2)	-0.31667	-0.21667	-0.21667	-0.11667	-0.13333	-0.583333	
	23,5<M2	-0.9	-0.8	-0.8	-0.5	-0.45	0	
	1,23,4 (M3)	-0.2375	-0.1875	-0.1875	-0.1875	-0.2125	-0.6625	
	1,4,5 <M1	-0.9	-0.8	-0.8	-0.5	-0.45	0	
π=12|3|4|5	min	-0.2375	-0.1875	-0.1875	-0.23333	-0.28333	-0.733333	π_3=12|345
	12,3 (M1)	-0.16667	-0.13333	-0.13333	-0.23333	-0.28333	-0.733333	
	12,4 < M1	-0.18333	-0.15	-0.15	-0.25	-0.26667	-0.716667	
	12,5 <M1	-0.9	-0.8	-0.8	-0.5	-0.45	0	
	3,4,5 <M1	-0.9	-0.8	-0.8	-0.5	-0.45	0	
	12,3,4 (M2)	-0.2375	-0.1875	-0.1875	-0.1875	-0.2125	-0.6625	
π=1|2|3|45	1,2,3*	-0.16667	-0.13333	-0.13333	-0.23333	-0.28333	-0.733333	π_2=12|34|5
	i,45;i=1,2,3	-0.9	-0.8	-0.8	-0.5	-0.45	0	
π=12|3|45	12,3*	-0.16667	-0.13333	-0.13333	-0.23333	-0.28333	-0.733333	π_2=12|34|5
	12,45	-0.9	-0.8	-0.8	-0.5	-0.45	0	
	3,45	-0.9	-0.8	-0.8	-0.5	-0.45	0	
π=1|23|45	1,23*	-0.16667	-0.13333	-0.13333	-0.23333	-0.28333	-0.733333	π_5=1|2|34|5
	1,45	-0.9	-0.8	-0.8	-0.5	-0.45	0	
	23,45	-0.9	-0.8	-0.8	-0.5	-0.45	0	
π=123|45	123*	-0.16667	-0.13333	-0.13333	-0.23333	-0.28333	-0.733333	π_2=12|34|5
π=123|4|5	123*	-0.16667	-0.13333	-0.13333	-0.23333	-0.28333	-0.733333	π_2=12|34|5
	123,4	-0.2375	-0.1875	-0.1875	-0.1875	-0.2125	-0.6625	
	123,5	-0.9	-0.8	-0.8	-0.5	-0.45	0	
π=1|2345	2345*	-0.4625	-0.3625	-0.3625	-0.2125	-0.2125	-0.4375	π_3=12|345
π=1234|5	1234*	-0.2375	-0.1875	-0.1875	-0.1875	-0.2125	-0.6625	π_4=1|2|345
π=12345	12345*	-0.37	-0.31	-0.31	-0.25	-0.26	-0.53	π_1=1|234|5
π=1|2|3|4|5	min	-0.31667	-0.25	-0.25	-0.23333	-0.28333	-0.733333	π_1=1|234|5
	1,2,3 (M1)	-0.16667	-0.13333	-0.13333	-0.23333	-0.28333	-0.733333	
	1,2,4 <M1	-0.18333	-0.15	-0.15	-0.25	-0.26667	-0.716667	
	i,j,5 for i,j<5	-0.9	-0.8	-0.8	-0.5	-0.45	0	
	2,3,4 (M2)	-0.31667	-0.21667	-0.21667	-0.11667	-0.13333	-0.583333	
	1,3,4 (M3)	-0.28333	-0.25	-0.25	-0.15	-0.16667	-0.616667	
	1,2,3,4 (M4)	-0.2375	-0.1875	-0.1875	-0.1875	-0.2125	-0.6625	

Table 1: Details of example 1. Pay offs U^i for selected partitions. "\|" signifies a party boundary,
M_i a potential government, "$*$" signifies uniqueness and "$< M_j$" domination by M_j via $\lhd$.
Parties in M_i are separated by commas. "min" signifies taking the minimum over pay offs for different M_i.
The final column notes possible deviations to other partitions.

Please select Sheet 2" for computations and definitions

Partition	Government	$U^1=U^2$	U2	U^3	U^4	U^5	**$U^6=U^m$**	$U^7=U^8$	$U^9=U^{10}=U^{11}$
π*=1|234567|8|9….	**234567***	**-0.12108**	**-0.12108**	**-0.07175**	**-0.05375**	**-0.05375**	**-0.05408**	**-0.10442**	**-0.12892**
	1,234567	-0.11064	-0.11064	-0.06179	-0.05407	-0.0605	**-0.06093**	-0.11486	-0.13936
	234567,8	-0.208	-0.208	-0.158	-0.104	-0.059	**-0.058**	-0.0175	-0.042
π_1=12345|67|89 10 1	**12345,67***	**-0.2045**	**-0.2045**	**-0.1545**	**-0.1005**	**-0.0555**	**-0.0545**	**-0.021**	**-0.0455**
π_2=1|2345|67|89 10	**min**	**-0.20508**		**-0.15508**	**-0.10108**	**-0.05608**	**-0.05508**	**-0.021**	**-0.0455**
	1,2345,67 (M1)	-0.2045	-0.2045	-0.1545	-0.1005	-0.0555	**-0.0545**	-0.021	-0.0455
	2345,67 (M2)	-0.20508	-0.20508	-0.15508	-0.10108	-0.05608	**-0.05508**	-0.02042	-0.04492
π_3=1|2345|678|9|10	**min**	**-0.208**		**-0.158**	**-0.104**	**-0.059**	**-0.058**	**-0.01838**	**-0.04288**
	1,2345,678*	-0.20713	-0.20713	-0.15713	-0.10313	-0.05813	**-0.05713**	-0.01838	-0.04288
	2345,678*	-0.208	-0.208	-0.158	-0.104	-0.059	**-0.058**	-0.0175	-0.042
π_4=1|23456|7|8|9| 1	**1,23456***	**-0.0755**	**-0.0755**	**-0.05883**	**-0.05883**	**-0.07383**	**-0.0745**	**-0.15**	**-0.1745**
	23456,7	-0.20508	-0.20508	-0.15508	-0.10108	-0.05608	**-0.05508**	-0.02042	-0.04492
	1,23456,7	-0.2045	-0.2045	-0.1545	-0.1005	-0.0555	**-0.0545**	-0.021	-0.0455
π_5=1|2345|6|7|8|9|	1,23456*	-0.0755	-0.0755	-0.05883	-0.05883	-0.07383	**-0.0745**	-0.15	-0.1745
	23456,7	-0.13629	-0.13629	-0.08629	-0.06314	-0.05671	**-0.05686**	-0.08921	-0.11371
	1,23456,7	-0.1255	-0.1255	-0.0755	-0.062	-0.062	**-0.06225**	-0.1	-0.1245
π_6=1|2|345678|9|…	**345678***	**-0.15067**	**-0.15067**	**-0.10067**	**-0.06467**	**-0.04967**	**-0.04967**	**-0.07483**	**-0.09933**
	2,345678	-0.13629	-0.13629	-0.08629	-0.06314	-0.05671	**-0.05686**	-0.08921	-0.11371
π_7=1|234|5678|..	**min**	**-0.208**	**-0.208**	**-0.158**	**-0.104**	**-0.059**	**-0.058**	**-0.01838**	**-0.04288**
	234,5678	-0.208	-0.208	-0.158	-0.104	-0.059	**-0.058**	-0.0175	-0.042
	1,234,5678	-0.20713	-0.20713	-0.15713	-0.10313	-0.05813	**-0.05713**	-0.01838	-0.04288
π_8=1|234|567|8|..	**min**	**-0.208**	**-0.208**	**-0.158**	**-0.104**	**-0.05993**	**-0.06036**	**-0.11429**	**-0.13879**
	1,234,567 (M1)	-0.11121	-0.11121	-0.06121	-0.0535	-0.05993	**-0.06036**	-0.11429	-0.13879
	234,567,8 (M2)	-0.208	-0.208	-0.158	-0.104	-0.059	**-0.058**	-0.0175	-0.042
	1,234,567,8 (M3)	-0.20713	-0.20713	-0.15713	-0.10313	-0.05813	**-0.05713**	-0.01838	-0.04288
π_9=1|234|56|78|..	**min**	**-0.20917**	**-0.20917**	**-0.15917**	**-0.10517**	**-0.06017**	**-0.05917**	**-0.13333**	**-0.15783**
	1,234,56 (M1)	-0.09217	-0.09217	-0.04217	-0.04217	-0.05717	**-0.05783**	-0.13333	-0.15783
	1,234,56,78 (M2)	-0.20713	-0.20713	-0.15713	-0.10313	-0.05813	**-0.05713**	-0.01838	-0.04288
	234,56,78 (M3)	-0.20917	-0.20917	-0.15917	-0.10517	-0.06017	**-0.05917**	-0.01633	-0.04083
π_{10}=1|23|4567|8|..	**min**	**-0.208**	**-0.208**	**-0.158**	**-0.104**	**-0.05993**	**-0.06036**	**-0.11429**	**-0.13879**
	1,23,4567 (M1)	-0.11121	-0.11121	-0.06121	-0.0535	-0.05993	**-0.06036**	-0.11429	-0.13879
	23,4567,8 (M2)	-0.208	-0.208	-0.158	-0.104	-0.059	**-0.058**	-0.0175	-0.042
	23,4567 (M3)	-0.12142	-0.12142	-0.07142	-0.05342	-0.05342	**-0.05375**	-0.10408	-0.12858
π_{11}=123|456|7|8|9…	**min**	**-0.20713**	**-0.20713**	**-0.15713**	**-0.10313**	**-0.05813**	**-0.0585**	**-0.134**	**-0.1585**
	123,456 (M1)	-0.0915	-0.0915	-0.04283	-0.04283	-0.05783	**-0.0585**	-0.134	-0.1585
	123,456,7 (M2)	-0.2045	-0.2045	-0.1545	-0.1005	-0.0555	**-0.0545**	-0.021	-0.0455
	123,456,7,8 (M3)	-0.20713	-0.20713	-0.15713	-0.10313	-0.05813	**-0.05713**	-0.01838	-0.04288
π_{12}=12345|6|7|89 10	**min**	**-0.208**	**-0.208**	**-0.158**	**-0.104**	**-0.059**	**-0.058**	**-0.13333**	**-0.15783**
	12345,6 (M1)	-0.09217	-0.09217	-0.04217	-0.04217	-0.05717	**-0.05783**	-0.13333	-0.15783
	12345,6,7 (M2)	-0.208	-0.208	-0.158	-0.104	-0.059	**-0.058**	-0.0175	-0.042
π_{13}=123|456|7|89 10	**123,456***	**-0.0915**	**-0.0915**	**-0.04283**	**-0.04283**	**-0.05783**	**-0.0585**	**-0.134**	**-0.1585**
	123,456,7	-0.2045	-0.2045	-0.1545	-0.1005	-0.0555	**-0.0545**	-0.021	-0.0455
π_{14}=12345|678|9|10	**12345,678***	**-0.20713**	**-0.20713**	**-0.15713**	**-0.10313**	**-0.05813**	**-0.05713**	**-0.01838**	**-0.04288**
π_{15}=123|45678	**123,45678***	**-0.1255**	**-0.1255**	**-0.0755**	**-0.062**	**-0.062**	**-0.06225**	**-0.1**	**-0.1245**
π_{16}=1234|5678	**1234,5678***	**-0.20713**	**-0.20713**	**-0.15713**	**-0.10313**	**-0.05813**	**-0.05713**	**-0.01838**	**-0.04288**
π_{17}=123|4|56|78	**min**	**-0.208**	**-0.208**	**-0.158**	**-0.104**	**-0.059**	**-0.058**	**-0.13333**	**-0.15783**
	123,4,56 (M1)	-0.09217	-0.09217	-0.04217	-0.04217	-0.05717	**-0.05783**	-0.13333	-0.15783
	123,4,56,78 (M2)	-0.20713	-0.20713	-0.15713	-0.10313	-0.05813	**-0.05713**	-0.01838	-0.04288
	123,56,78 (M3)	-0.208	-0.208	-0.158	-0.104	-0.059	**-0.058**	-0.0175	-0.042
	123,56 < M1	-0.0898	-0.0898	-0.0398	-0.0506	-0.0596	**-0.0602**	-0.1357	-0.1602
π_{18}=1234|56|78	**min**	**-0.20713**	**-0.20713**	**-0.15713**	**-0.10313**	**-0.05813**	**-0.05783**	**-0.13333**	**-0.15783**
	1234,56 (M1)	-0.09217	-0.09217	-0.04217	-0.04217	-0.05717	**-0.05783**	-0.13333	-0.15783
	1234,56,78 (M2)	-0.20713	-0.20713	-0.15713	-0.10313	-0.05813	**-0.05713**	-0.01838	-0.04288

Table 2: Details of example 2. Pay offs U^i for selected partitions. "\|" signifies a party boundary,

M_i a potential government, "$*$" signifies uniqueness and "$< M_j$" domination by M_j via $\lhd$.

Parties in M_i are separated by commas. "min" signifies taking the minimum over pay offs for different M_i.

please select "Sheet 2" for computations, definitions and further partitions