# Intra-party decision making, party formation, and moderation in multiparty systems*

Gerald Pech†

October 8, 2011

**Abstract**

We establish coalitional stable party structures of a party formation game in an elected assembly. Farsighted political players can commit to form parties and to vote on policies according to the party position which is determined by intra-party majority rule. Parties may form governments and block proposals by a randomly selected member of the government. If the government recognition rule allows for the formation of multiparty governments, the median parliamentarian either realizes her ideal point or a policy lottery which she strictly prefers to the status quo. This outcome is enforced by the threat of forming a moderating centre party.

**Keywords:** Endogenous political parties, intraparty decision rule, farsighted coalitional stability, political institutions.

**JEL codes:** C72, D71, D78.

*A first draft of this paper was written while I was May Wong-Smith fellow at the University of St Andrews. Financial support is gratefully acknowledged. For helpful comments I wish to thank the associate editor, two anonymous referees, Ruvin Gekker, Indridi Indridason, Laszlo Koczy, Hideo Konishi, Nadeem Naqvi, Ashley Piggins and seminar participants at the APSA meeting in Boston, LGSIII in Siena, CEPET 2004 in Udine, Games 2004 in Marseille, MPSA 2005 in Chicago, PET 2005 in Marseille, the Mid-West Economic Theory workshop 2005 in Kansas and SCW 2008 in Montreal.

†KIMEP, e-mail: gp@geraldpech.net

# 1 Introduction

This paper develops a model of endogenous party formation in an elected assembly in which parties form governments and governments select a policy by a random proposal maker process. In the last stage, the policy selection stage, a proposal is voted against the status quo. Each party coordinates its votes by intra-party majority rule but within a multiparty government, all parties have to accept the proposal. This is compatible with a view of parties taking a role in negotiating policy outcomes in multiparty governments and enforcing them in a parliamentary vote. At the same time each party is constrained by the need to win majority support among its own parliamentarians in the collective decision processes within the party. Seen from an ex ante perspective, with these assumptions a government generates a policy lottery. At the government formation stage, governments form for a given party structure. For the party formation stage, I establish stable party structures of a party formation game applying the largest consistent set (Chwe, 1994) as solution concept. I show that with a multiparty selection rule, under which a coalition of parties commanding an absolute majority is selected as government, the median parliamentarian realizes either her ideal point or a policy lottery which she strictly prefers to the status quo. Such an outcome is remarkable because the median parliamentarian is not pivotal in all governments which may form in stable party structures and may have to settle for worse outcomes than the status quo in some votes. Moreover, a multiparty government which preserves the status quo is conceivable, although such a government does not form in equilibrium unless the status quo coincides with the median voter's ideal point. In the end, it is the option of forming a moderating centre party where the median voter is pivotal which allows us to eliminate all party structures where the median parliamentarian does not improve over the status quo.

I compare the multiparty selection rule to a majoritarian selection rule where a party with plurality in parliament is selected as government. This rule has effects which are comparable to multiparty selection provided that a parliamentarian majority is allowed to block proposals irrespective of party membership. Here, the paper relates to the debate, mainly regarding the US congress, on whether parties are effective in shaping policy outcomes and on the relative importance of the legislative median versus the median in the majority party in the legislative process.[1] In Europe, where multiparty government is common, parties are typically identified as unitary actors which negoatiate political outcomes between them and can to some extent discipline the various factions of which they are composed (see e.g. Laver/Schofield 1990).

Our results shed some light on empirical regularities found in different parliamentary systems. Multiparty systems are often characterized by small centrist parties (Schofield/Sened, 2006). Huber and Powell (1994) who distinguish the ideal forms of majority control and proportional influence systems - the latter being identified with more elements of power-sharing - find that government policies in proportional influence systems tend to be closer to the position of the median voter compared to majority control systems. This form of policy convergence is in spite of the fact that party positions in multiparty systems do typically not converge to the political center (Schofield/Sened, 2006). Birchfeld and Crepaz (1999) find that systems of proportional representation (PR) are characterized by more redistribution relative to systems with plurality voting and relate their finding to the fact that under power sharing arrangements typical of PR systems the consent of more political players is required for accepting a policy proposal.[2]

This paper endogenizes the emergence of parties as veto players in an assembly under multiparty selection.[3] What drives the formation of parties is the non-proportional relationship between positive agenda power, i.e. the power to set the agenda, and the size of the group formed which is typical of parliamentary institutions: For example, the biggest parliamentarian faction may be asked to form the government, a party or a coalition of parties may have to pass an inauguration with an overall majority to become the government,[4] chairs in committees may go to the largest party. On the other hand, by joining a party agents may sacrifice negative agenda power, i.e. the power to block proposals, because they accept to vote with the party majority.[5]

[1]See Cox/McCubbins (2005) and Krehbiel (2007). Diermeier/Vlaicu (2011) propose a unifying theory which obtains the Krehbiel result in the limit.

[2]They argue that when players interact in one assembly, they are under greater pressure to produce more responsive policies, that is policies oriented towards the median voter. In this paper, by contrast, veto players endogenously emerge and, as a consequence, policies are bent towards the preferences of the median parliamentarian.

[3]See Tsebelis (1995) on the effect of exogenous veto points in spatial games.

[4]See Diermeier/Eraslan/Merlo (2003) for an overview of rules of government formation.

[5]Parties have a range of means to force compliance of their legislators (see Cox, 2006). Within our more confined model, one way of ensuring support of potential dissenters

Forming a small party increases negative agenda power provided that it is indispensable as member in a government. In particular, in a small centrist party politicians are able to block more proposals than if they had organized themselves within a larger party. Here, this paper suggests a rationale for the formation of small centrist parties with ideologically motivated politicians. This is different from the conventional wisdom which suggests that small centre parties form because they enable office seeking politicians to increase the probability of entering the government.

Whilst we do not model the electoral stage and instead focus on the formation of parties in an elected assembly, the results still have obvious relevance for the formation of parties which form in order to win elections: It seems reasonable to assume that only such parties can credibly contest an election, which are stable against defections after the election is concluded.[6]

## 1.1 Related literature

Only recently, a number of papers have begun to analyze the emergence of parties as coalitions of political agents. Austen-Smith (2000) combines party formation and policy selection. He models parties as representing endogenously formed economic classes and finds that policies with more redistribution are obtained under proportional representation but that policies under plurality rule are closer to the median's preferences. Morelli (2004) determines effective parties and policies under proportional representation and majority voting. His setting involves multiple districts with a multitude of candidates running in each district. Bandyopadhyay/Oak (2008) present a citizen candidacy model in which groups of citizens select a candidate/party, and parties form governments. In Osborne/Tourky (2004) agents organize themselves in parties, committing themselves to vote for the same position. With economies of scale and with single-issue politics a two-party system emerges, irrespective of the electoral system. Kaminski (2006) focuses on the effect of electoral laws on party structure and hence on the stability of electoral rules and political parties. In his model parties are the movers which may select platforms, split or coalesce. If a coalition commands a majority in

would be to tie the vote on policy to a vote of no confidence, which is not uncommon in parliamentary democracies. Voting against the proposal would result in dissolution of parliament and legislators would be threatened with losing any benefits which are tied to their parliamentary seat.

[6] I am grateful to a referee for suggesting this interpretation.

the chamber, it may change the electoral rule. Levy (2004) models platform selection where credible choices are restricted to the Pareto set of the party members. She shows that there are strategy choices which parties can adopt which cannot be matched by individual candidates.

There are a number of models where parties emerge as a response to an environment which creates uncertainty. In the coalition bargaining model of Jackson/Moselle (2003), legislators join parties to ensure that they are not excluded from a winning coalition. In Eguia (2011a, b), legislators are uncertain about their true preferences but hold priors regarding their probability to vote for or against a proposal. In a stylized example, Eguia (2011a) shows that some players want to commit to vote with their party and form voting blocs whilst members in the middle of the spectrum refuse to commit. Similarly, in the present paper it is in the face of random proposal maker selection that parties allow legislators to realize preferred lotteries.

In a theory of party formation, parties have to change the set of equilibrium policy outcomes (Krehbiehl, 1993). Eguia (2011 a,b), as does this paper, focuses on ex ante incentives for parliamentarians to commit their votes.[7] In the bargaining model of Jackson/Moselle, a parliamentarian who has joined a party can be guaranteed her continuation pay off from reverting to a game without parties. The agenda setting power of parties and costly bargaining affect the policy outcome. As Diermeier/Vlaicu (2011) show, institutions which support unequal agenda setting power may arise in the equilibrium of an organizational game in parliament.

The present paper models parties as coalitions of political agents and describes the emergence of parties as an equilibrium phenomenon. But rather than purely focusing on party formation it explains, as Austen-Smith (2000) and, with a different focus, Kaminski (2004) do, party structures as they are characteristic of parliamentary democracies with proportional influence systems and identifies their common effect on policy outcomes. For this analysis, we require a solution concept which is sufficiently inclusive, i.e. which contains a rich class of potentially stable party structures and at the same time sets an intuitively convincing threshold for eliminating outcomes. In addition, we wish to employ a solution concept which achieves this end without imposing too many restrictions on agents' moves, acknowledging that party formation potentially involves complex coalitional bargaining processes. The

[7] Eguia (2011b) discusses the case where the party has small enforcement power and enforces party discipline only when no parliamentarian is pivotal.

solution concept which we found most appealing given these criteria is the largest consistent set (Chwe, 1994).[8] This solution does not preclude any coalitional move on which the agents involved unanimously agree. It does presume farsighted agents, i.e. agents which are deterred from a deviation which might result in an outcome which does not represent an improvement over their current situation. Thus, agents are assumed to behave conservatively, i.e. they have a predilection to the status quo situation. This assumption ensures existence of a solution and it specifies what we mean by a stable party structure.

In the end, all models of party formation have to deal with the issue of how to ensure stability in an environment, which naturally gives rise to moves by coalitions of agents. Levy (2004) also presumes farsightedness on the side of agents but applies equilibrium binding agreements as solution concept (Ray and Vohra, 1997). There, coalitional moves are restricted to deviations which result in finer, not coarser, coalition structures. Kaminski considers a myopic coalitional equilibrium which is proof against splits and mergers. Eguia considers unilateral deviations and, in his application, split-proof equilibria, i.e. equilibria which are proof against further splits of parties when agents are myopic.

As we are interested in combining party formation and government formation, forward-looking behavior is a natural assumption to make. At the party formation stage, farsightedness implies that parliamentarians take into account moves by other parliamentarians which are precipitated by their own move. This behavioral assumption is meaningful in a model of party formation. For example, the four Labour Party defectors which founded the UK Social Democratic Party, two of them sitting Members of Parliament, would have thought about which further members they would attract from the remaining Labour party (as they eventually did).

I set up the model in section 2. Section 3 states the main results. Section 4 discusses the implications of our modeling decisions for our results.

[8] We show that the concept of equilibrium binding agreements generally makes different predictions. However, in the case of multiparty selection it supports similar qualitative results.

# 2 The model

The set of members of the assembly is $N$. Their number $n$ is finite, odd and $n \geq 3$. We restrict ourselves to the case where the policy space is $X = [0, 1]$. The reason for doing so is that we want to cast our results in terms of the policy preferences of the median voter. Even this simple case gives rise to a rich political environment. A member of the assembly $i$ has an ideal point $\widehat{x}^i \in X$ and a utility $u_i = v(|x - \widehat{x}^i|)$ which decreases in the distance of the policy realization $x$ from the ideal point according to a concave function $v$. For simplicity, we assume that politicians can be strictly ordered according to their ideal points. This assumption is not too restrictive as ideal points may be arbitrarily close. As a consequence, any player can be identified by her ideal point $\widehat{x}^i$.

## 2.1 Party structures and inducement correspondences

We aim at deriving stable party structures where we identify a party structure with a partition $\pi$ of the set of players $N$. We focus on party structures where parties are connected coalitions of politicians, i.e. comprise of agents with consecutive ideal points.[9]

**Definition 1** *A party structure $\pi = \{P_1, \ldots, P_J\}$ is a partition of $N$ into parties $P_i$. $\pi$ is exhaustive, i.e. $\cup_{i=1}^J P_i = N$, each $P_i \in \pi$ is consecutive, i.e. for any three agents with $\widehat{x}^a < \widehat{x}^b < \widehat{x}^c$ if $a \in P_i$ and $c \in P_i$ then $b \in P_i$, and no parties overlap, i.e. $P_i \cap P_j = \emptyset$ for all $P_i, P_j \in \pi, P_i \neq P_j$.*

We denote $\Pi$ the set of all partitions satisfying definition 1. Because parties are consecutive, a party $P$ can be identified by its member with the lowest and highest preference, $\widehat{x}_{P^l}$ and $\widehat{x}_{P^u}$. For adjacent parties $P_i$ and $P_{i+1}$ the ideal points of $\widehat{x}_{P_i^h}$ and $\widehat{x}_{P_{i+1}^l}$ are also adjacent and can be ranked consecutively, which we indicate by writing $\widehat{x}_{(P_i^u)+1} = \widehat{x}_{P_{i+1}^l}$ and $\widehat{x}_{P_i^u} = \widehat{x}_{(P_{i+1}^l)-1}$.

A coalition $S$ can modify a partition by forming a party which previously was not there. If a coalition which is at the centre of a party breaks away this leaves the two wings of the previous party and the deviating coalition each as separate parties so as to satisfy definition 1.

[9]For a justification see Axelrod (1970). The role of this condition is to allow us to rank party medians.

**Definition 2** *A consecutive coalition* $S = \{\widehat{x}_{S^l}, .., \widehat{x}_{S^u}\}$ *can replace the party structure* $\pi = \{P_1, .., P_J\}$ *with party structure* $\{S, \pi_{-S}\} = \{P_1, .., \underline{P}_i, S, \overline{P}_j, .., P_J\}$ *where* $\underline{P}_i = \{\widehat{x}_{P^l i}, ..., \widehat{x}_{(S^l)-1}\}$ *and* $\overline{P}_j = \{\widehat{x}_{(S^u)+1}, ..., \widehat{x}_{P_j^u}\}$.

We describe what coalition $S$ can do by using the inducement correspondence "$\xrightarrow{S}$" and write $\pi \xrightarrow{S} \pi'$ where it is understood that $\pi' = \{S, \pi_{-S}\}$.

## 2.2 Recognition Rules and Potential Governments

Given $\pi$, parties form a government. For each $\pi$ and given the recognition rule we determine the set of governments $G(\pi)$ which are "likely" to form at the government formation stage:

Under the multiparty recognition rule, parties may further commit and form ties between them. The singleton party or group of parties which commands an absolute majority of votes in the assembly necessary to pass an inauguration vote becomes the government. We assume that one such government always forms. First, we define the set of governments which meet the recognition rule:

**Definition 3** *(Multiparty recognition rule) Let* $M_i = \{P_1, ..., P_K \in \pi\}$ *be a collection of parties such that their joint membership yields an absolute majority in the assembly. The union of all such collections,* $\Omega(\pi) = \bigcup_i M_i(\pi)$, *is the set of multiparty governments meeting the recognition rule given* $\pi$.

Whilst there might be many multiparty governments which satisfy the recognition rule, we want to focus on those governments $G(\pi) \subseteq \Omega(\pi)$ which are "likely" to form. We maintain that any government might form which is not dominated by any other government in terms of the pay off expectation $U^i$ it creates for a majority of each of its constituent parties' members. Because we cannot exclude a priori the possibility, that this dominance relationship is cyclical, we also include all governments which are included in a cycle according to the dominance relationship. Our motivation for doing so is that forming coalition governments is a matter of bargaining between the parties in parliament and, whilst we are agnostic about the precise circumstances under which this bargain takes place, it is reasonable that the final outcome should be picked amongst those candidates.

**Definition 4** *(Potential governments under the multiparty recognition rule) A government $M_a$ is dominated by a government $M_b$, in short $M_b \rhd M_a$, if the expected pay off under $M_b$, $U^i(M_b)$, exceeds the expected pay off under $M_a$, $U^i(M_a)$, for a majority of agents $i$ of every $P \in M_b$. Let $\blacktriangleright$ denote the transitive closure of relation $\rhd$, that is $M_b \blacktriangleright M_a$ if there is a finite sequence $M_b \rhd M_{a_n} \rhd M_{a_{n-1}}, .., M_{a_1} \rhd M_a$. The set of potential governments given $\pi$ is $G(\Omega(\pi), \rhd) = \{M_j \in \Omega | M_k \in \Omega$ and $M_k \blacktriangleright M_j$ imply $M_j \blacktriangleright M_k\}$.*

This notion of potential governments corresponds to the admissible set (Kalai/Schmeidler, 1977) of the government formation game. It is well-known that the admissible set is non empty and, therefore, $G(\pi)$ is non-empty. In the case where $\rhd$ itself is transitive $G(\pi)$ consists of those elements which are not strictly dominated via $\rhd$.[10]

The majoritarian recognition rule selects the party with plurality in the assembly as the government party. If there is a tie, one of the tied parties is selected by a tie-breaking mechanism. This defines the set of potential governments $G(\pi)$ under the majoritarian recognition rule.

**Definition 5** *(Potential governments with majoritarian recognition) Under majoritarian recognition, $G(\pi)$ is the set of parties with plurality in the assembly.*

## 2.3 Pay offs

Each player uniquely associates with each party structure $\pi$ a pay off, i.e. there is a mapping $\Pi \to (U)^n$. To obtain this mapping, we construct the policy lottery $\ell_M$ associated with government $M$ as follows: Let $S_M = \bigcup_{P \in M} P$ be the set of parliamentarians who are member of a government party. Each $i \in S_M$ has an equal chance to be selected as proposal maker. The proposal becomes government policy if it finds the necessary majorities in parliament as specified by the parliamentary institution $I$. In the multiparty selection system, majorities are needed in all government parties. For our discussion of majoritarian selection we focus on a system of checks and balances where a proposal has to be accepted by the government party and by a simple majority of parliamentarians. If a proposal fails to find the necessary majorities, the status quo policy $x^{sq} \in X$ is realized by default. Let $D(x^{sq}, M, \pi, I)$ be

[10] $\rhd$ is asymmetric as the governments which are compared have at least one party and the majorities in that party at least one parliamentarian with strict preferences in common.

the set of proposals which find the majorities specified by $I$ when the status quo policy is $x^{sq}$, the government $M$ and the party structure $\pi$. An agent, when indifferent, approves of a proposal, hence $D$ is compact. By lemma 1, it is also convex:

**Lemma 1** *The set $D(x^{sq}, M, \pi, I)$ is convex.*

**Proof.** See appendix A. ■

A rational proposal for parliamentarian $i \in S_M$ is $p^i = \arg\max[u_i(x)$ s.t. $x \in D(x^{sq}, M, \pi, I)]$. By convexity of $D$ and single-peakedness of $u$, $p^i$ is unique. With equal selection probabilities, $\Pr(p^i) = 1/\#S_M$ and we get

$$\ell_{M_j} = (p^i(x^{sq}, M, \pi, I), 1/\#S_M))_{i \in S_M}.$$

Let $F^M(x) = Pr(p^i \leq x)$ be the distribution function associated with this lottery. The expected pay off of agent $k$ with government $M$ is $U^k(\ell_M) = \int u_k(x) dF^M(x)$.

One partition $\pi$ may give rise to more than one potential government, that is the set $G(\pi)$ defined in definition 4 for the multiparty selection rule and in definition 5 for the majoritarian selection rule might not be a singleton. Therefore, in order to assign $U(\pi)$ we need to aggregate pay offs over all governments $M \in G(\pi)$. We consider two aggregation rules. Under a pessimistic rule an agent assigns to the partition $\pi$ the pay off corresponding to her lowest utility under any government which may form, i.e. $U^k(\pi) = \inf\limits_{M \in G(\pi)} (U^k(\ell_M))$.[11] Under the aggregation rule corresponding to the Laplace criterion an agent assigns a uniform prior to all $M \in G(\pi)$ and associates partition $\pi$ with the compound lottery $\ell(G(\pi))$. Both aggregation rules have been advanced for decision situations under Knightian uncertainty where agents cannot assign a unique distribution of probabilities to the different outcomes. This is particularly plausible, when the outcome, that is the formation or investiture of a particular government, is the result of bargaining between parties or is for an outside arbiter to decide.

## 2.4 Stable party structures

Letting $\pi_a \succ_i \pi_b$ if and only if $U^i(\pi_a) > U^i(\pi_b)$, we derive a strict order $\{\succ_i\}$ on the finite set of partitions $\Pi$. Together with the inducement correspon-

[11] As a consequence, different agents may assign to partition $\pi$ pay offs associated with different governments.

dence $\{\xrightarrow{S}\}_{S\subset N, S\neq\emptyset}$ this defines a party formation game $\Gamma = (N, \Pi, \{\succ_i\}_{i\in N}, \{\xrightarrow{S}\}_{S\subset N, S\neq\emptyset})$. The solution concept we apply in solving the game assumes agents to be farsighted, i.e. when they contemplate a deviation from a partition $\pi$ they take into account deviations by other coalitions subsequent to their own deviation. An agent or a coalition $S$ of agents might be willing to deviate, if a partition reached after subsequent deviations, such as $\pi_b$, is better for all deviators $i \in S$ than the partition $\pi_a$ they deviated from or, in short, if $\pi_b \succ_S \pi_a$. Before saying which partitions are stable, we note that the partitions which *might* be reached from an initial partition $\pi_a$ include all those partitions $\pi_b$ for which subsequent coalitions of farsighted agents could be persuaded to undertake coalitional moves which finally bring about $\pi_b$. These partitions indirectly dominate $\pi_a$:

**Definition 6** *(Indirect dominance) A partition $\pi_a$ is indirectly dominated by partition $\pi_b$, or $\pi_a \ll \pi_b$, if there is a sequence $\pi_k \xrightarrow{S_k} \pi_{k+1}$, for $k = 1, ...K$ with $\pi_1 = \pi_a$ and $\pi_{K+1} = \pi_b$ and $\pi_b \succ_{S_k} \pi_k$ for all $S_k$.*

The set of partitions which are feasible when the status quo partition is $\pi$ consists of $\pi$ itself and those partitions which indirectly dominate $\pi$:

$$\Xi(\pi) \equiv \{\pi\} \cup \{\pi' \in \Pi | \pi \ll \pi'\}$$

It is the attractiveness of deviations to partitions which are stable or from which agents could conceivably move to ultimately stable positions which exclude a partition from the solution. Chwe (1994) defines a set $Y$ as consistent if for all $\pi_a \in Y$ every deviation is deterred. A deviation to some partition $\pi_d$ is deterred as long as there is at least one stable partition which might be reached, i.e. which is in $\Xi(\pi_d) \cap Y$, which the deviating coalition does not prefer to the original partition $\pi_a$:

**Definition 7** *A set $Y \subset \Pi$ is consistent if $\pi_a \in Y$ if and only if $\forall \pi_d$, $S$ such that $\pi_a \xrightarrow{S} \pi_d$, $\exists \pi_e \in Y$, where $\pi_d = \pi_e$ or $\pi_d \ll \pi_e$, such that $\pi_a \not\prec_S \pi_e$.*

The largest consistent set of a game, $LCS(\Gamma)$, is the union of all consistent sets. Chwe (1994, proposition 1) shows that for every game $\Gamma$ there uniquely exists a largest consistent set. The following proposition is due to Chwe (1994).

**Proposition 1** *(Chwe, 1994) The largest consistent set of* $\Gamma$ *is non empty. Furthermore, it has the external stability property, i.e. for all* $\pi \in \Pi \backslash LCS(\Gamma)$, *there is* $\pi' \in LCS(\Gamma)$ *such that* $\pi \ll \pi'$.

**Proof.** Because $\Pi$ is finite there are no infinite $\ll$-chains, so $\Gamma$ satisfies the condition of proposition 2 in Chwe (1994). Hence $LCS(\Gamma)$ is non empty and satisfies the external stability property. ■

# 3 Results

Before I go on deriving stable party structures for the multiparty recognition rule I establish a result which allows us to derive comparative individual rankings of the policy lotteries generated by each government. We can derive such a ranking for lotteries which are comparable by first order stochastic dominance, only using the assumption that agents have a common evaluation of distances, $v$. The latter assumption is weaker than the condition for deriving majority preferences over lotteries (Banks/Duggan, 2006).

**Lemma 2** *For any two lotteries* $\ell^a$ *and* $\ell^b$ *where* $\ell^b$ *stochastically first order dominates* $\ell^a$ *and two agents* $h$ *and* $l$ *with* $\widehat{x}^h < \widehat{x}^l$*: (a) if* $h$ *prefers* $\ell^b$ *over* $\ell^a$ *then* $l$ *also prefers* $\ell^b$ *over* $\ell^a$*; (b) if* $l$ *prefers* $\ell^a$ *over* $\ell^b$*, then* $h$ *does.*

**Proof.** See appendix B. ■

## 3.1 Stable partitions with multiparty selection

At the policy selection stage, a proposal needs majority approval in all parties in government when voted against the status quo policy $x^{sq}$. We assume $x^{sq} \geq \widehat{m}$, the median parliamentarian's ideal point. If $m$ runs as a singleton, she enforces a partition in the set $\Pi_m \subset \Pi$. In all such partitions every government in $G(\pi)$ offers a lottery which is strictly better for $m$ than the status quo or equally good if $x^{sq} = \widehat{m}$:

**Lemma 3** *Let* $\Pi_m$ *be the set of partitions where* $m$ *runs as a singleton. With multiparty selection, in every* $\pi \in \Pi_m$ $m$ *realizes a better pay off than with* $(x^{sq}, 1)$ *or, if* $x^{sq} = \widehat{m}$*, her ideal point.*

**Proof.** See appendix C. ∎

Lemma 3 gives a rationale for the emergence of small centrist parties. Because $m$ can enforce a preferred party structure, no party structure is stable unless she improves on $(x^{sq}, 1)$ or achieves her bliss point. This result is independent of the aggregation rule (i.e. pessimism or Laplace).

**Proposition 2** *In every stable party structure with multiparty selection, $m$ realizes a better pay off than with $(x^{sq}, 1)$ or, if $x^{sq} = \widehat{m}$, her ideal point.*

**Proof.** Consider $x^{sq} > \widehat{m}$ and a partition $\pi \notin \Pi_m$ such that $M \in G(\pi)$ with lottery $\ell_M \not\succ_m (x^{sq}, 1)$. $\pi \notin LCS(\Gamma)$: There exists a deviation $\pi \xrightarrow{m} \pi'$, $\pi' \in \Pi_m$. By lemma 3, $\pi' \succ_m \pi$ for all $\pi' \in \Pi_m$. Because $m$'s cooperation is needed in moving away from $\Pi_m$, also for all $\pi'' \in \Xi(\pi') \backslash \Pi_m$, $\pi'' \succ_m \pi'$. By proposition 1, $\Xi(\pi) \cap LCS(\Gamma) \neq \emptyset$. Hence, $\pi \notin LCS(\Gamma)$. In the case $x^{sq} = \widehat{m}$ and $\pi \in \Pi_m$, all $M \in G(\pi)$ offer $(\widehat{m}, 1)$. ∎

It is worthwhile noting the significance of lemma 2 for this result: Given $\pi \in \Pi_m$, a government $M$ which induces a lottery which leaves $m$ no better off than the status quo is dominated by some government $M_0$ which is comparable by stochastic dominance. Because of the stochastic dominance relation, $M_0$ must either exclude $M$ or $M$ and $M_0$ must be excluded from $G$ by some other government.[12] The following corollary characterizes the set of possible policy realizations in a stable party structure:

**Corollary 1** *Let $P_k$ be the right-most party in $M$ and $m_k$ the agent which turns a minority in $P_k$ into a majority when counting from below. Define $x'_k$ the smallest $x$ which leaves $m_k$ indifferent to $x^{sq}$. In a stable partition $\pi$, the set of implementable policies for government $M \in G(\pi)$ is $Z_M \subseteq D$ with $D = [x'_k, x^{sq}]$.*

**Proof.** See part D of the appendix ∎

Because proposition 2 makes only a statement about expected pay offs, it is natural to ask, whether policies which are worse for $m$ than the status quo can never be observed as a final outcome. Our answer is negative:

**Remark 1** *In a stable party structure, $x \in Z_M$ and $x \prec_m x^{sq}$ is possible.*

[12] The assumption that only consecutive coalitions may form parties is necessary for this property to hold. Otherwise, $m$ would not be able to prevent on her own a partion $\pi'$ with a party $W$ whose members' ideal points are on both sides of $m$'s and whose party median satisfies $\widehat{m}_W < \widehat{m}$.

Appendix E provides the outline of an example of a stable partition with a potential one-party government where an agent $\widehat{x}_i < \widehat{m}$ is pivotal in blocking.[13] Once this government has formed, proposals which are worse for $m$ than the status quo can get a majority within the government party.

## 3.2 Stable partitions with majoritarian selection and checks and balances in parliament

This paper developed a theory where "veto points" at the policy selection stage emerge endogenously at the party formation stage under the multiparty selection rule. As a consequence, the median parliamentarian does better than with the status quo. In this section we compare this result to a majoritarian selection procedure under which the party with plurality is selected as government.[14] It is easy to construct examples, where stable partitions exist under majoritarian selection in which the median voter loses compared to the status quo. Therefore, we consider majoritarian rule together with a system of checks and balances at the policy selection stage where a proposal from the party with plurality not only requires the approval of a majority of party members, but also a majority in parliament. This introduces an additional exogenous "veto point". The following proposition characterizes stable party structures under this institution in analogy to proposition 2.

**Proposition 3** *In every stable party structure under a majoritarian selection rule with checks and balances in parliament $m$ realizes a pay off which is strictly greater than with $(x^{sq}, 1)$ or, if $x^{sq} = \widehat{m}$, her ideal point.*

**Proof.** See appendix F. ■

Any partition which leaves $m$ indifferent to $(x^{sq}, 1)$ is unstable because there exists a deviation which includes $m$ and all agents on one side of the median by which all deviators gain.

[13] We were not able to construct an example with fewer than 11 players. Also, the example is sensitive to small changes in the parameters.

[14] In majority control systems it is typically the aggregation of votes at the level of electoral districts, not the aggregation of votes in parliament which is governed by plurality rule. As a rule for selecting the government, plurality rule allows us to assign a non empty set of potential governments to each partition. Imposing an absolute majority requirement would force us to arbitrarily assign an outcome to partitions for which the set of potential governments is empty.

Trivially, the set of acceptable policies under checks and balances is $D = [x', x^{sq}]$ where $x' = x < x^{sq}$ such that $m$ is indifferent to $x^{sq}$. Whilst under multiparty selection the set of acceptable policies may be strictly included in $[x', x^{sq}]$, the reverse case cannot be ruled out as shown by remark 1. There are no general results on the relative performance of the institutions which is unsurprising because under majoritarian selection governments may include fewer agents than a simple majority.

Finally, consider a scenario where under multiparty selection a proposal needs acceptance by a majority of party members in each government party and, in addition, a majority in parliament where no party discipline is imposed. Unsurprisingly, the result of proposition 2 still applies in this case. It is worthwhile pointing out that incentives to form a moderating centre party still exist in this scenario: A coalition of agents can commit to block policy proposals - i.e. not to put forward proposals which do not have a majority in all government parties - which some of them would have accepted individually because an agent's rationality dictates that she accepts any proposal which offers at least her status quo utility.

# 4 Discussion

This section discusses our modeling decisions and their role in obtaining the results. Agents are farsighted and can commit to form parties and to adhere to intra-party majority rule but they cannot commit to policies. Some degree of farsightedness is indispensable if we want to coherently model two decision stages. Also, some compromise on policy commitments is unavoidable in multiparty democracy because parties bargain over policy positions after elections. If we had replaced intra party majority rule for policy choices by party unanimity, i.e. the requirement that policy proposals are unanimously accepted by the members of each party in government, unifying or subdividing parties for a given government would not change policy outcomes. In this case, a multiparty government would not distinguish itself from a single party government comprised of different factions rather than parties.

We model party formation in an elected assembly. The model could be extended to an environment where citizen-candidates may form parties which enter the electoral race. Electoral competition with sincere voters and policy motivated politicians would tend to erode the party landscape.[15]

[15]Pech (2010) shows that in every partition in the farsighted stable set of a citizen-

As solution concept we applied the largest consistent set. The largest consistent set may be large. In the remainder of this section we consider strong Nash equilibrium and equilibrium binding agreements as alternative solution concepts.

## 4.1 Strong Nash Equilibrium

Chwe (1994) shows that any strong Nash equilibrium of an appropriately defined coalitional contingent threat situation (Greenberg, 1990) in which deviations are strictly deterred is also consistent. However, as the following counter example demonstrates, for our party formation game with multiparty selection strong Nash equilibrium does not in general exist.

Let a player's strategy consist in the announcement $c_i$ of which party he or she joins and, following Konishi, Le Breton and Weber (1997), let $\pi(c)$ be a partition or party structure by chosen strategies, i.e. if $i \in S \in \pi(c)$, then for any $j \in S$ $c_i = c_j$ and for any $k \notin S$ $c_i \neq c_j$. Hence, $\times_i C_i \longrightarrow \Pi$ and pay offs corresponding to a strategy profile $c$ are $U^i(c) = U^i(\pi(c))$. A profile $c \in C$ is a strong Nash equilibrium if for all $S \subset N$ and for all $d_S \in C_S \backslash \{c_S\}$ there exists $j \in S$ such that $U^j(c) \geq U^j(d_S, c_{N \backslash S})$. We insist on unanimity in party formation, hence we restrict strategies for a deviating coalition such that $i \in S$ and $k \in N/S$ imply $d_i \neq c_k$.[16]

Corresponding to the coalitional threat situation we can define a game $\Gamma = (N, \Pi, \{\prec_i\}_{i \in N}, \{\xrightarrow{S}\}_{S \subset N, S \neq \emptyset})$ where $\pi(c) \xrightarrow{S} \pi'(e)$ if $c_{N \backslash S} = e_{N \backslash S}$ and $d_i \in e_S$ $c_i \in e_{N \backslash S}$ implies $d_i \neq c_i$ (Chwe, 1994).

**Example 1** *Consider a game with* $N = \{1, 2, 3, 4, 5\}$ *and* $u_i(x) = -|x - \widehat{x}^i|$, *with the distribution of bliss points* $\widehat{x}^1 = 0.1$, $\widehat{x}^2 = 0.2$, $\widehat{x}^3 = 0.5$, $\widehat{x}^4 = 0.55$, $\widehat{x}^5 = 1$ *and* $x^{sq} = 1$. *Agents are pessimistic.*[17]

Partition $\pi_1 = \{\{1\}, \{2, 3, 4\}, \{5\}\}$ results in $M = \{2, 3, 4\}$ and gives the pay off vector $(-.3167, -.2167, -.1167, -.1333, -.5833)$, partition $\pi_2 =$

candidacy model with firm pre-electoral coalitions, the party of the median voter runs as a singleton and wins the election. A farsighted stable set is also consistent (see Chwe, 1994).

[16] Using this standard definition we allow for deviations by $S$ which do not affect coalitions $T \subset N \backslash S$ even if connectedness of $T$ is broken. It turns out that this slight change in the definition of what a deviating coalition can do does not affect our results concerning the existence of strong Nash equilibria.

[17] For details, see the online repository accomanying this paper (attached).

$\{\{1,2\},\{3,4\},\{5\}\}$ results in $M = \{\{1,2\},\{3,4\}\}$ and gives $(-.2375, -.1875, -.1875, -.2125, -.6625)$, partition $\pi_3 = \{\{1,2\},\{3,4,5\}\}$ results in $M = \{3,4,5\}$ and gives $(-.1667, -.1333, -.2333, -.2833, -.7333)$ as does partition $\pi_4 = \{\{1\},\{2\},\{3,4,5\}\}$ and partition $\pi_5 = \{\{1\},\{2\},\{3,4\},\{5\}\}$ gives $(-.3167, -.2167, -.2333, -.2833, -.7333)$. These partitions dominate every other partition in $\Pi$, i.e. there is a deviation by a coalition which results in $\pi_1$, $\pi_2$, $\pi_3$, $\pi_4$ or $\pi_5$. Yet none of these partitions is stable: Coalition $\{1,2\}$ wants to deviate from $\pi_1$ and $\pi_5$ and induces $\pi_2$, coalition $\{3,4,5\}$ wants to deviate from $\pi_2$ and induces $\pi_3$. Coalition $\{2,3,4\}$ wants to deviate from $\pi_3$ and $\pi_4$, inducing $\pi_1$.

## 4.2 Equilibrium binding agreements

A partition or party structure $\pi$ is an equilibrium binding agreement (EBA) if there is no sequence of credible deviations through which a party structure $\pi'$ is reached which represents a binding agreement and in which the leading perpetrator gains (Ray/Vohra, 1997). A sequence of deviations is credible, if any re-merging of other coalitions which participate in this sequence is undone by a sequence of deviations ending in $\pi'$ with one of the other coalitions as a leading perpetrator. Furthermore, only subcoalitions of existing coalitions are allowed to form. In our context this implies that an emerging party cannot recruit members across parties. We call the coarsest party structure which is an EBA an equilibrium party structure. Levy (2004) uses the concept of equilibrium binding agreements to derive party structures.

Our exposition follows Diamantoudi/Xue (2007). Define $\Gamma = (N, \Pi, \{\overset{S}{\longmapsto}\}_{S\subset N, S\neq\emptyset}, \{\succ_i\}_{i\in N})$ where $\pi \overset{S}{\longmapsto} \pi'$ implies that $S \subset T$ for some $T \in \pi$ and $S$, $\underline{T} = \{x_{T^l}, ..., x_{(S^l)-1}\}$ and $\overline{T} = \{x_{(S^u)+1}, ..., x_{T^u}\} \in \pi'$.[18] A partition $\pi_b$ RV-dominates $\pi_a$, i.e. $\pi_b \gg_{R\&V} \pi_a$ if

1. there is a sequence $\pi_k \overset{S_k}{\longmapsto} \pi_{k+1}$ for $k = 1, ..., K$ with $\pi_a = \pi$ and $\pi_K = \pi_b$,
2. there is a leading perpetrator $S_1$ for which $\pi_b \succ_{S_1} \pi_a$,
3. suppose that some $\pi_c$ can be formed by re-merging some of the other (connected) perpetrators $S_2, ..., S_K$. $\pi_c$ is $RV$-dominated by $\pi_b$ with one of the coalitions $S_2, ..., S_K$ as the leading perpetrator.

[18]That any unconnected remainder of a coalition falls into two parties is a disgression from Ray/Vohra's definition of what a perpetrator can do.

Using RV-dominance, equilibrium binding agreement can be redefined as the von Neumann-Morgenstern stable set $V$ of $(\Pi, \gg_{R\&V})$.[19] Then $\pi$ is an $EBA$ if and only if $\pi \in V$. The coarsest coalition structure in $V$ is referred to as an equilibrium party structure ($EPS$) of the game. The set $EPS(\Gamma)$ uniquely exists and is non-empty. It is straightforward to show that a proposition analogous to proposition 2 applies to equilibrium party structures:

**Proposition 4** *In every equilibrium party structure with multiparty selection, $m$ either realizes a better pay off than with $(x^{sq}, 1)$ or, if $x^{sq} = \widehat{m}$, her ideal point.*

**Proof.** Suppose the status quo partition is $\pi \precsim_m (x^{sq}, 1)$. If $m$ deviates as leading perpetrator some $\pi' \in \Pi_m$ is reached. As all partitions which RV-dominate $\pi'$ are in $\Pi_m$, by lemma 3 $m$ is better off than with $(x^{sq}, 1)$. If $x^{sq} = \widehat{m}$, all governments in $\Pi_m$ offer $\ell = (\widehat{m}, 1)$. ■

In part G of the appendix, we show that proposition 3 does not extend to the case of equilibrium binding agreements, i.e. partitions where $m$ fares as well as with the status quo may be an $EPS$. Hence, there exists $\Gamma$ and partitions which are in $EPS(\Gamma)$ but not in $LCS(\Gamma)$. As we show in part H of the appendix, there also exists $\Gamma$ for which there are partitions which are in $LCS(\Gamma)$ but not in $EPS(\Gamma)$. Hence, neither solution includes the other.

# 5 Appendix

## 5.1 Appendix A: Proof of lemma 1:

Let $D^i(x^{sq})$ be the set defined by $\{x | x \succsim_i x^{sq}\}$ which is convex. $D(S, x^{sq})$, the join of $D^i$ for a majority of members of $S$ is convex: If $x^a \in D$ and $x^b \in D$, $x^c = \lambda x^a + (1 - \lambda)x^b \in D$. Suppose that not: there must exist $i$, $j \in S$ with $x^b \succ_i x^{sq}$, $x^{sq} \succ_i x^a$ and $x^a \succ_j x^{sq}$, $x^{sq} \succ_j x^b$. In the case, $x^a < x^b < x^{sq}$, who prefers $x^a$ to $x^{sq}$ also prefers $x^b$, contradicting the stipulated order for $i$ and $j$. So say, $x^a < x^{sq} \leq x^b$. If $x^a < x^c \leq x^{sq}$ ($x^c > x^{sq}$), then $x^c$ gets at least as many votes as $x^a$ ($x^b$) contradicting $x^a \in D$ ($x^b \in D$). For two parties $S$ and $T$, $D(S, T, x^{sq})$ is the join of convex sets $D(S, x^{sq})$ and $D(T, x^{sq})$ which is convex.

[19] In the case of nested deviations the predictions of the original formulation of $EBA$ and $V$ coincide.

## 5.2 Appendix B: Proof of lemma 2

Let the cumulative distribution function $F : X \to [0,1]$ represent a lottery. Its utility is $U = \int u(x)dF(x)$. The lottery $\ell^b$ stochastically first order dominates $\ell^a$ if their cumulative distribution functions satisfy $F^b(x) \leq F^a(x)$ for every $x$. $U(\ell^b) \geq U(\ell^a)$ if $\int u(x)dH(x) \geq 0$ for $H(x) = F^a(x) - F^b(x) \leq 0$ (see Mas-Colell/Whinston/Green, 1995, proposition 6.D.1). Integrating by parts we obtain $\int u(x)dH(x) = [u(x)H(x)]_0^1 - \int u'(x)H(x)dx$. The first term yields zero because $H(1) = 0$ and $H(0) = 0$.

We now have to show that for two agents $l$ and $h$ with ideal-points $\widehat{x}^l > \widehat{x}^h$, if $-\int u_h'(x)H(x)dx > 0$ then $-\int u_l'(x)H(x)dx > 0$ and if $-\int u_l'(x)H(x)dx < 0$ then $-\int u_h'(x)H(x)dx < 0$. With the absolute value of $u'$ only depending on $|x - \widehat{x}^i|$ we have: For $x \leq \widehat{x}^h$, $0 \leq u_h'(x) \leq u_l'(x)$. For $\widehat{x}^h < x \leq \widehat{x}^l$, $u_h'(x) < 0$ and $u_l'(x) \geq 0$. For $x > \widehat{x}^l$, $u_h'(x) \leq u_l'(x) < 0$. Collecting arguments, $-\int u_h'(x)H(x)dx \leq -\int u_l'(x)H(x)dx$ from which both claims follow.

## 5.3 Appendix C: Proof of lemma 3

Let $\widehat{m}_k$ be the ideal point of the median of party $P_k$ if $\#P$ is odd or, if $\#P$ is even, the ideal point of the agent who turns a minority into a majority when counting from below. Let $\underline{\widehat{m}}_k$ be the ideal point of the agent who does the same when counting from above. $\underline{\widehat{m}}_k \leq \widehat{m}_k$ with equality if $\#P$ is odd.

**Lemma 4** *Say $x^{sq} \geq \widehat{m}$. (a) If a government $M$ includes as its right-most party some $P_k$ with $\widehat{m} \leq \widehat{m}_k < x^{sq}$ , then $\ell_M \succ_i (x^{sq}, 1)$ for all $i \leq m$. (b) If $\widehat{m}_k \geq x^{sq}$ and there is a party $j$ with $\underline{\widehat{m}}_j \leq x^{sq}$ in $M$, $M$ offers $\ell_M = (x^{sq}, 1)$.*

**Proof.** Define $x_k' = x < x^{sq} | x \sim_{m_k} x^{sq}$ for median $m_k$.To see that part (a) is true note that proposals $p$ satisfy $x_k' \leq p \leq x^{sq}$ or are blocked by $m_k$ or $\underline{m}_k$, respectively. Thus, $m$ weakly prefers every $p$ to $x^{sq}$ and strictly prefers $p = m_k$ from which $\ell_M \succ_m (x^{sq}, 1)$ follows. Because $(x^{sq}, 1)$ stochastically dominates $\ell_M$, the claim also holds for all $i$ with $\widehat{x}^i < \widehat{m}$ by lemma 2 (a). Part (b) holds because with $\widehat{m}_k \geq x^{sq}$, $p < x^{sq}$ are blocked and with $\underline{\widehat{m}}_j \leq x^{sq}$, all $p > x^{sq}$ are blocked. ■

Lemma 3 in the main text states that in every $\pi \in \Pi_m$, the set of partitions where $m$ runs as a singleton, she receives a pay off which she strictly prefers to the status quo. In the following proof of the lemma, we have to take into account that the domination relationship between governments $\rhd$ is not necessarily transitive.[20]

**Proof.** Let $\pi \in \Pi_m$ be a partition where $m$ runs as a singleton. Let $\{P_1, .., P_J, m\}$ be a partition of $S = \{i | \widehat{x}_i \leq \widehat{m}\}$. Clearly, $M_0 = \{P_1, ..., P_J, m\}$ $\in \Omega(\pi)$, the set of selectable governments.

If $x^{sq} = \widehat{m}$, all $M \in \Omega(\pi)$ offer $(\widehat{m}, 1)$. So consider $x^{sq} > \widehat{m}$. By lemma 4(a), all governments in $\Omega(\pi)$ for which the right-most $P_k$ satisfies $\widehat{m} \leq \widehat{m}_k < x^{sq}$, including $M_0$, offer $m$ a better lottery than $(x^{sq}, 1)$. By lemma 4(b), all governments $\Omega(\pi)$ with $\widehat{m}_k \geq x^{sq}$ offer the lottery $(x^{sq}, 1)$. Say, $M_l$ is such a government. By lemma 2 (a), for all $\widehat{x}_i \leq \widehat{m}$, $\ell_{M_0} \succ_i \ell_{M_l}$ and $M_0 \rhd M_l$. Suppose $M_l \in G(\pi)$:[21]

First, consider $M_0 \notin G(\pi)$. In that case, by definition 4, $M_k \blacktriangleright M_0$ must be true and $M_0 \blacktriangleright M_k$ false for some $M_k \in G(\pi)$. But from the first claim with $M_0 \rhd M_l$ follows $M_k \blacktriangleright M_l$. By definition 4, $M_l \in G(\pi)$ only if $M_l \blacktriangleright M_k$. But $\blacktriangleright$ being the transitive closure, this implies $M_0 \blacktriangleright M_k$ contradicting $M_0 \notin G(\pi)$.

Next, consider $M_0 \in G(\pi)$. Because $M_0 \rhd M_l$, by definition 4, $M_l \in G(\pi)$ only if $M_l \blacktriangleright M_0$. This implies there is $M_k$ with $M_l \rhd M_k$. Because by lemma 4(b), for $m$ an outcome cannot be worse than $(1, \tau^{sq})$, $M_k$ must offer a (weakly) better lottery. If $\ell(M_k) \succsim_m (x^{sq}, 1)$ we have $\ell(M_k) \succsim_i (x^{sq}, 1)$ for all $i$ with $\widehat{x}^i \leq \widehat{m}$ by lemma 2 (b), because $(x^{sq}, 1)$ stochastically dominates $\ell(M_k)$. In order to dominate via $\rhd$, at least one party $P_j$ must be included in $M_l$ where an agent $\underline{m}_j$ with $\widehat{x}^i \leq \widehat{m}$ is decisive and for whom $(x^{sq}, 1) \succ_i \ell(M_k)$, a contradiction. Therefore, $M_l \notin G(\pi)$. ■

## 5.4 Appendix D: Proof of corollary 1

The lower boundary of $D$ follows from the definition of $x'_k$. Upper boundary: Suppose there is stable $\pi$, such that some $M$ admits $p > x^{sq}$. By lemma 4 (b), proposals $p > x^{sq}$ are blocked unless all parties $j$ in $M$ have $\widehat{\underline{m}}_j > x^{sq}$. In that case all proposals satisfy $p \geq x^{sq}$ with strict inequality for some proposals.

[20]Banks and Duggan (2006) provide an example where a lottery is preferred by an opposing pair of extreme voters

[21]To proof our claim for the case of a pessimistic aggregation rule, we have to show that $M_l \in G(\pi)$ results in a contradiction. For the case of the Laplace aggregation rule, $m$ assings the same pay off to $\pi$ as under $(1, x^{sq})$ if all $M \in G(\pi)$ offer the lottery $(x^{sq}, 1)$. Again, to rule out this possibility it is sufficient show that $M_l \in G(\pi)$ results in a contradiction.

Hence, $(x^{sq}, 1) \succ_m \ell_M$, which by proposition 2 contradicts stability of $\pi$.

## 5.5 Appendix E: Proof of remark 1

In this section we construct an example supporting remark 1.

**Example 2** *Consider a game with* $x^{sq} = 0.6$, $n = 11$ *and* $u_i(x) = -|x - \widehat{x}^i|$ *with the distribution of bliss points* $\widehat{x}^1 = 0.35$, $\widehat{x}^2 = 0.35 + \varepsilon$, $\widehat{x}^3 = 0.4$, $\widehat{x}^4 = 0.454$, $\widehat{x}^5 = 0.499$, $\widehat{x}^6 = \widehat{m} = 0.5$, $\widehat{x}^7 = 0.5755$, $\widehat{x}^8 = 0.5755 + \varepsilon$ *and* $\widehat{x}^i = 0.6 + (i - 9)\varepsilon$ *for* $i = 9, 10, 11$, $\varepsilon \to 0$. *Agents are pessimistic.*

Suppose that partition $\pi^* = \{\{1\}, S, \{8\}, \{9\}, ..\}$ with $S = \{2, 3, 4, 5, 6, 7\}$ has formed: In $S$ agent 5 is pivotal with a bliss point on the left hand side of the median in parliament, agent 6. Because the status quo is $x^{sq} = 0.6$, agent 5 is indifferent between $x = 0.398$ and the status quo. Agent 2 proposes 0.398 and this proposal is accepted by a majority of agents in $S$. With this proposal the median in parliament fares worse than with the status quo. With this partition, $S$ is the only element in the set of potential governments. Overall, the median voter realizes an expected utility of $-0.054083$ which is better than the status quo pay off of $-0.1$.

Details of the example are provided in an online repository which accompanies this paper. There it is shown that $\pi^*$ is stable. Other stable partitions are $\{\{1, 2, 3, 4, 5\}, \{6, 7\}, \{8\}, \{9\}, \{10\}, \{11\}\}$ and $\{\{1, 2\}, \{3, 4, 5, 6, 7, 8\}, \{9\}, \{10\}, \{11\}\}$. In the former $m$ does worse than in $\pi^*$ but in the latter, $m$ and all agents on her right hand side do better. Yet deviations by any subset of those agents are deterred.

## 5.6 Appendix F: Proof of proposition 3

Ignoring the trivial case $x^{sq} = \widehat{m}$, assume $x^{sq} > \widehat{m}$ and define $x' = x < x^{sq} | x \sim_m x^{sq}$. Recall that coalitions are connected. Define $S = \{i | \; \widehat{x}_i \leq \widehat{m}\}$ and $T = \{i | \; \widehat{x}_i \geq \widehat{m}\}$. We assume that $S' \in G$ offers $\ell_{S'}$ such that $(x^{sq}, 1) \sim_m \ell_{S'}$, the case where $T' \subset T$ offers such a lottery is similarly analyzed. Because no agent can pass a proposal $p^i \prec_m x^{sq}$ and $m$ proposes $\widehat{m} \neq x^{sq}$ this implies $m \notin S'$.

a) Suppose that in $\pi'$ $S' \subseteq S$ and $T' \subseteq T$ draw and that $U^m(\pi') = U^m(x^{sq}, 1)$. For this to hold under Laplace it must be that the associated lotteries are $\ell_{S'} = (1, x')$ and $\ell_{T'} = (1, x^{sq})$ and, hence, $m \notin S'$, $T'$. Under pessimism, the condition must be fulfilled at least for one of $S'$ and $T'$ which by assumption is $S'$. Now there is a deviation to $\pi''$ with $T = \{m, ...., n\}$ winning and offering $\ell''$. It is immediate that $\ell'' \succ_T (1, x^{sq})$ under pessimism. Because in $\ell''$ $m$ proposes $\widehat{m}$ with positive probability, $U^i(\ell'') > U^i(0.5\ell_{S'} + 0.5\ell_{T'})$ for all $i \in T$ under Laplace.

Consider $\pi''' \in \Xi(\pi'')$ and suppose that $U^m(\pi''') = U^m(x^{sq}, 1)$. Then $m$ cannot be member of a winning coalition. If $T'' \subset T$ is winning, there must be a sequence from $\pi''$ to $\pi'''$ with $\pi^k$ and $\pi^k \xrightarrow{R} \pi^{k+1}$, with $R \cap S \neq \emptyset$ when in $\pi_k$ $T$ was winning or $T'$ was drawing. But because $(x^{sq}, 1)$ is the worst outcome for $i \in S$, $\pi''' \not\succ_R \pi_k$, contradicting $\pi''' \in \Xi(\pi'')$.

Suppose $S'' \subset S$ is winning in $\pi'''$. Then there must be a sequence from $\pi''$ to $\pi'''$ with $\pi^k$ and $\pi^k \xrightarrow{R} \pi^{k+1}$, with $R \cap T \neq \emptyset$ when at $\pi^k$ $T$ was winning. Because $\ell_T$ stochastically dominates $\ell_{S''}$, this implies that at least $m$ must prefer $\ell_{S''}$ and hence $\ell_{S''} \succ_m \ell_T \succsim_m (x^{sq}, 1)$.

Suppose there is a draw in $\pi'''$ and that $U^m(\pi''') = U^m(1, x^{sq})$. In that case, the distribution of pay offs is the same as in $\pi''$. But then there is a sequence from $\pi''$ to $\pi'''$ with $\pi^k$ and $\pi^k \xrightarrow{R} \pi^{k+1}$, with $R \cap T \neq \emptyset$ when at $\pi^k$ $T$ was winning. As we have seen, all $i \in T$ prefer $\pi''$.

b) Suppose that in $\pi'$, $S'' \subset S'$ wins and offers $(x', 1)$. Again there is a deviation by $T$ to $\pi''$. $\ell_T$ stochastically dominates $(x', 1)$ and $\ell_T \succ_T (x', 1)$. As above, we can show that in every $\pi''' \in \Xi(\pi'')$ $m$ must improve on $(x', 1)$.

c) Next suppose that in $\pi'$, $S'' \subset S'$ is drawing and that $\pi' \sim_m (x^{sq}, 1)$. Again there is a deviation by $T$ to $\pi''$. This case is analyzed in part a) where we note that $\ell_T$ stochastically dominates $\ell_{S''}$ and, if $m$ prefers $\ell_T$, $\ell_T \succ_T \pi'$.

In all cases, $\forall \pi \in \Xi(\pi'')$ $\pi \succ_m (x^{sq}, 1)$. By proposition 1, $\Xi(\pi'') \cap LCS \neq \emptyset$. Hence no $\pi'$ with $\pi \sim_m (x^{sq}, 1)$ is stable.

## 5.7 Appendix G

We construct an example where in an $EPS$ under majority selection with checks and balances, the median voter does no better than under the status quo: Specify $n = 13$, $u_i(x) = -|x - \widehat{x}^i|$ and $\widehat{x}^i = 0 + \varepsilon(i-1)$ for $i \leq 5$, $\widehat{x}^6 = 0.5 - \varepsilon$, $\widehat{x}^7 = \widehat{m} = 0.5$, $\widehat{x}^i = 0.5 + \varepsilon(i-7)$ for $i = 8, ..., 11$, $\widehat{x}^{12} = 1 - \varepsilon$, $\widehat{x}^{13} = 1$. Suppose that $\varepsilon \to 0$, $x^{sq} = 0.9$ and agents are pessimistic. Consider the partition $\pi_1 = \{\{1,2,3,4,5\}, \{6,7,8,9\}, \{10,11,12,13\}\}$ in which $\{1,2,3,4,5\}$ wins. All proposals in $\{1,2,3,4,5\}$ satisfy $p^i \sim_m x^{sq}$. No deviation occurs, hence $\pi_1$ is an $EBA$. Find a partition which is coarser by merging coalitions and show that it is not an $EBA$. Partition $\{\{1,2,3,4,5\},\{6,7,8,9,10,11,12,13\}\}$ is not in $V$ because it is $R\&V$-dominated by $\pi_2 = \{\{1,2,3,4,5\},\{6,7,8,9,10,11\},\{12,13\}\}$. $\pi_2$ is not coarser than $\pi_1$. Similarly, we exclude the grand coalition and the partition $\{\{1,2,3,4,5,6,7,8,9\},\{10,11,12,13\}\}$. Thus $\pi_1$ is an $EPS$.

## 5.8 Appendix H

In this section we construct a partition $\pi \in LCS(\Gamma)$ which is not in $EPS(\Gamma)$.[22] It is necessary to find a partition in $EPS(\Gamma)$ which blocks $\pi$ by a nested deviation but is itself not stable, that is blocked by some other partition in $LCS(\Gamma)$: Specify $n = 9$, $u_i(x) = -|x - \widehat{x}^i|$, $\widehat{x}^i = 0.4 + (i-1)\varepsilon$ for $i \leq 4$, $\widehat{m} = 0.5$,$\widehat{x}^6 = 0.5 + \varepsilon, \widehat{x}^7 = 0.5 + 2\varepsilon$, $\widehat{x}^8 = 1 - \varepsilon$, $\widehat{x}^9 = 1$. The status quo is $x^{sq} = 0.9$, $\varepsilon \to 0$, agents use the Laplace aggregation rule and the multiparty selection rule applies.

Partitions $\pi_1 = \{1,2,3,4\},\{5,6,7\},\{8,9\}\}$ and $\pi_2 = \{\{1,2,3\},\{4,5,6,7\},\{8,9\}\}$ yield identical pay offs[23] and are both consistent. It is easy to see that any subdivision of $\{1,2,3,4\}$ or $\{5,6,7\}$ results in a positive probability of a party with identical interest being omitted from government in some $M \in G$ such as in $\pi' = \{\{1,2,3\},\{4\},\{5,6,7\},\{8,9\}\}$ with $G = \{(\{1,2,3\},\{4\},\{5,6\}),(\{1,2,3\},\{5,6\})\}$. The first potential government yields $\ell = (0.1, \Pr = 4/7; 0.5, \Pr = 3/7)$ and the second $\ell = (0.1, \Pr = 1/2; 0.5, \Pr = 1/2)$. With Laplace aggregation rule, $\ell(\pi') = (0.1; \Pr = 15/28; 0.5, \Pr = 13/28)$. A deviation by $\{5,6,7\}$ from $\pi_2$ to $\pi'$ is deterred by $\{1,2,3,4\}$ moving to $\pi_1$.

If, however, only nested deviations are possible, $\{5,6,7\}$ want to deviate from $\pi_2$ to $\pi'$. Thus, $\pi^2$ is not in $EPS(\Gamma)$ whilst $\pi_1$ is (note that there is no coarser $EBA$).

[22] For details of this example please refer to the separate sheet provided for verification purposes.

[23] The unique sets of potential governments are $G(\pi_1) = (\{1,2,3,4\},\{5,6,7\})$ and $G(\pi_2) = (\{1,2,3\},\{4,5,6,7\}$. In both cases, is lottery is $\ell = (0.1, \Pr = 4/7; 0.5, \Pr = 3/7)$ .

## References

[1] Austen-Smith, D., 2000. Proportional Representation versus Majority Voting. Journal of Political Economy 108: 1235-1266.

[2] Axelrod, Robert, 1970. Conflict of Interest: A Theory of Divergent Goals with Application to Politics. Chicago, Markham, 1970.

[3] Bandyopadhyay, S., Oak, M., 2004. Party Formation and Political Bargaining: A Model of Proportional Representation, FEEM Working Paper No. 98.2004.

[4] Banks, J.S., Duggan, J., 2006. A social choice lemma on voting over lotteries with applications to a class of dynamic games. Social Choice and Welfare 26: 285-304.

[5] Birchfeld, V., Crepaz, M.M.L., 1998. The Impact of Constitutional Structures and Collective and Competitive Veto Points on Income Inequality in Industrialized Democracies. European Journal of Political Research 34: 175-200.

[6] Chwe, M., 1994. Farsighted Coalitional Stability. Journal of Economic Theory 63: 299-325.

[7] Cox, G.W., 2006, The Organization of Democratic Legislatures, in: B. Weingast and D. Wittman (eds.), The Oxford Handbook of Political Economy, Oxford University Press.

[8] Cox, G.W., McCubbins, M.D., 2005. Setting the Agenda: Responsible Party Government in the House of Representatives. Cambridge University Press.

[9] Diamantoudi, E., Xue, L., 2007. Coalitions, Agreements and Efficiency. Journal of Economic Theory 136: 105-125.

[10] Diermeier, D., Eraslan, H., Merlo, A., 2003. A structural model of government formation. Econometrica 71: 2770.

[11] Diermeier, D., Vlaicu, R., 2011. Parties, Coalitions, and Internal Organization of Legislatures, American Political Science Review 105: 359-380.

[12] Eguia, J., 2011a. Endogenous Parties in an Assembly. American Journal of Political Science 55: 16-26.

[13] Eguia, J., 2011b. Voting Blocs, Party Discipline, and Party Formation, Games and Economic Behavior 73: 111-135.

[14] Greenberg, J., 1990. The Theory of Social Situations, Cambridge University Press.

[15] Huber, J.D., Powell Jr, B.G., 1994. Congruence between Citizens and Policy Makers in Two Versions of Liberal Democracy. World Politics 46: 291-326.

[16] Jackson, M.O., Moselle, B., 2002. Coalition and Party Formation in a Legislative Voting Game, Journal of Economic Theory 103: 49-87.

[17] Kalai, E., Schmeidler, D., 1977. An Admissible Set Occuring in Various Bargaining Situations, Journal of Economic Theory 14: 402-411.

[18] Kaminski, M.M., 2006. A general equilibrium model of multi-party competition. Social Choice and Welfare 26: 336-361.

[19] Konishi, H., Le Breton, M., Weber, S., 1997. Equivalence of Strong and Coalition-Proof Nash Equilibria in Games without Spillovers, Economic Theory 9: 97-113.

[20] Krehbiel, K., 1993, Where's the Party? British Journal of Political Science 23: 235-266.

[21] Krehbiel, K., 2006. Partisan Roll Rates in a Nonpartisan Legislature, The Journal of Law, Economics and Organisization 23: 1-23.

[22] Levy, G., 2004. A model of political parties. Journal of Economic Theory 115: 250-277.

[23] Mas-Colell, A., 1995. Whinston, M.D., Green, J.R., Microeconomic Theory, Oxford University Press.

[24] Morelli, M., 2004. Party Formation and Policy Outcomes under Different Electoral Systems. Review of Economic Studies 71: 829-53.

[25] Osborne, M.J., Tourky, R., 2008. Party Formation on Single-Issue Politics, Journal of the European Economic Association 6: 974-1005.

[26] Pech, G., 2010. A Median Voter Theorem for Proportional Representation Systems, mimeo.

[27] Ray, D., Vohra, R., 1997. Equilibrium Binding Agreements, Journal of Economic Theory 73: 30-78.

[28] Schofield, N., Sened, I., 2006. Multiparty Democracy. Elections and Legislative Politics, Cambridge University Press.

[29] Tsebelis, G., 1995. Decision Making in Political Systems: Veto Players in Presidentialism, Parliamentarism, Multicameralism and Multipartyism, British Journal of Political Science 25: 289-325.

[30] Xue, L., 1997. Nonemptiness of the Largest Consistent Set, Journal of Economic Theory 73: 453-459.

[31] Xue, L., 1998. Coalitional Stability under Perfect Foresight, Economic Theory 11: 603-627.