Abstract

In a spatial model of political bargaining we show that there exists a constitution which a sufficiently patient autocrat would want to design and the parties forming a succeeding constitutional assembly would accept as a basis for negotiations on constitutional reform. That the middle class is opposed to redistribution strengthens the case for handing down a constitution and is a sufficient condition for the autocrat’s constitution to be efficient. Increases in middle class wealth make constitution writing more attractive unless taxation is “too effective” in redistributing wealth. We relate our findings to the so-called “Pinochet Constitution” in Chile.(98)

Keywords: Constitution, Political Transition, Legislative Bargaining, Redistribution, Inefficiency

JEL Classification: C72, D02, D72, H1
1 Introduction

Is there scope for an autocrat who wants to promote the interests of his constituency to select a constitutional status quo point, knowing that in succession it will ultimately fall to a democratically elected assembly to adopt a new constitution? In this paper we demonstrate that if the autocrat can offer a status quo point to the assembly and the alternative for the assembly is to enter an open bargaining process or conflict, the assembly can be persuaded to accept such a status quo point. Moreover, the autocrat can strategically exploit this situation and hand down a constitutional status quo point which serves the interests of his constituency. The reason why such a possibility exists is that from an ex ante point of view the conflict or open bargaining scenario results in a lottery. Players prefer certain outcomes and selecting a status quo point, even if it only serves as a starting point for further negotiations, narrows down potential outcomes of the constitutional bargaining process.

While this approach has potentially wide applicability\(^1\) our motivating example is Chile’s transformation from autocracy to democracy. This transition unfolded under the rules of the 1980 constitution which was promulgated by the Chilean junta. Following electoral defeat by president Pinochet, the Chilean parties of the center and the right negotiated constitutional amendments which were adopted by plebiscite as part of a reform constitution in 1989. In the process, the constitutional status quo was only marginally modified (see Barros, 2002, and Montes and Vial, 2005).

Chile fulfills conditions which are implicit or explicit to our paper and make successfully linking the autocratic constitution and the democratic constitution more likely: our analysis shows that a relatively wealthy middle class opposed to redistribution tends to make constitution writing more attractive to the autocrat and Chile largely fulfills this condition.\(^2\) Moreover, the situation which unfolded in Chile was such that the constitution handed down by the autocrat was a natural focal point for the constitutional assembly, a property we take for granted. Firstly, Chile had a long constitutional history before the military coup of 1973. Secondly, its “autocratic” constitution could actually be seen to constrain the junta: Pinochet, after an eight year transitory period, stood for re-election under “his” constitution and stood down as a consequence of his electoral defeat.

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\(^1\)See the case studies in Michalak and Pech (2013).

\(^2\)Chile has a relatively unfavorable Gini-index but, for most of its post-transition period, it has been governed by centrist governments and continued the neo-liberal reform agenda introduced under the Junta even after abolishing non-elected senators in the constitutional reforms of August 2005 which had been agreed between the right-wing Alliance and the center-left Concertacion. This outcome points to a preference of the median voter against redistribution.
This paper casts the autocratic constitutional choice problem in a spatial model of political bargaining. It extends the simple model of Michalak and Pech (2013) to take account of endogenous formation of bargaining coalitions. The formal model allows us to address some central issues in the study of constitutional stability and autocracy:

The view that a middle class which is interested in maintaining property rights is a prerequisite for constitutional stability is widely held.\(^3\) In the quite stylized setting of our model, we obtain the result that an increase in middle class wealth strengthens the autocrat’s incentives to write a constitution unless redistribution is ”too profitable” for a member of the benefited economic class.

Moreover, because autocracy is defined as the the capability to overturn outcomes of the institutionalized political process (Przeworski and Wallerstein, 1988), it is generally assumed that autocrats cannot commit.\(^4\) However, in our paper the autocrat voluntarily commits if he believes that abiding by his constitution is necessary to preserve its focal point character for a succeeding assembly.

1.1 Related Work

This paper is an exercise in positive constitutional analysis (see, e.g., Voigt, 1997 or Aghion, Alesina and Trebi, 2004). Our modeling approach builds on and expands advances in analysing the spatial bargaining model such as Baron, Diermeier and Fong (2012). In their dynamic, transferable utility framework, a patient agenda setter for a first period single-party government strategically selects an inefficient status quo policy point which gives her a bargaining advantage in the second period. The extent of the inefficiency is bounded by electoral considerations. Our model contrasts by not allowing transfers or a voting stage. We find that harmony of interests of middle class and the rich over tax rates is a sufficient condition for selecting an efficient status quo constitution, while the status quo constitution may be inefficient in the case of conflicting interests.

Michalak and Pech (2013) introduce a spatial model where the autocrat selects the constitutional status quo for exogenously given bargaining coalitions. In this paper we require the bargaining coalitions to form a strong Nash equilibrium. This allows us to provide an intuitive characterization of the constitutional choice of the autocrat who, when facing a middle class opposed to redistribution, wants to make prospective bargaining partners indifferent between negotiating with his constituency

\(^3\)See, for example, Ordeshook (1997) and Easterly (2001).

\(^4\)North and Weingast (1989) see the inability to commit not to change rules as a severe disadvantage to authoritarianism.
and with one another.

In a model of conflict with outside challengers, Myerson (2008) explores the conditions under which an autocrat wants to create institutions in order to commit to reward his supporters. Di Maria, Lazarova and Pech (2018) explore an autocratic sharing equilibrium with weak institutions.

Section 2 sets up the model. Section 3 introduces the reference model of de novo design of the constitution. Section 4 solves the constitutional choice problem of the autocrat in a static model of choice. Section 5 explores the effect of small changes of middle class wealth. Section 6 introduces the intertemporal model of constitutional choice. Section 8 discusses alternative modeling assumptions. Section 8 relates our results to the Chilean experience. Section 9 concludes.

2 A Model of Autocratic Constitutional Choice

We consider a model with three socio-economic groups\(^5\) which represent the main constituencies: The poor or the working class \(L\) favor redistribution, the middle class \(M\) may or may not prefer redistribution and the rich \(R\) object to redistribution. Their population shares are \(s_R, s_M, s_L, \sum s_i = 1\). The constituencies are represented by negotiating parties - of the left, the center and the right - which share their constituency’s preferences on social policy and have the same gross wealth \(w_R > w_M > w_L\). We assume that the right is the constituency of the autocrat who shares its preferences at the constitution writing stage.\(^6\)

In the absence of a written constitution, a process of de novo constitution writing unfolds after the demise of the autocrat. Crucially, we assume that the outcome of this process can only be predicted with some uncertainty. If there is a status quo constitution when the autocrat leaves office, representatives of the different political groups may decide to embark on a constitutional reform process based on the existing constitution. For this process to have sufficient credibility at least two of the three parties need to come together and negotiate the reform constitution.

The policy space is \(X \times T = \mathbb{R} \times [0, 1]\) with a redistribution dimension, represented by the tax rate \(t \in T\), and a social policy dimension \(x\), representing a country’s

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\(^5\)We do not explicitly model the military as a player. One may think of the military as a factor which inflicts on some parties a greater expected cost of freely negotiating the constitution.

\(^6\)In this we assume that the intertemporal agency problem between the elite and the autocrat has been resolved. Typically, autocracies are sustained by the exchange of economic advantages for support. Therefore, it is plausible that farsighted members of the elite are willing to trade support for favorable legislation at the constitutional level provided that they expect this legislation to serve their interest in the long run.
basic choices. x might be measured along a scale such as as liberalism versus authoritarianism, secularism versus Catholicism, or the relative importance of the social solidarity principle versus the free market principle.

A constitution c is a pair \((x, t)\). The utility function of a citizen belonging to class \(i\) is \(u_i = v_i(x) + w^n_i\) where \(w^n_i\) is citizen \(i\)'s net wealth after taxes and transfers and where \(v_i(x) = -\alpha_i |x - x_i|^2\) captures the loss associated with realizations on the social policy scale with \(x_i, i = L, M\), the bliss point of the respective group. We assume that \(x_M = 0, x_L = 1, \alpha_L = \alpha_M = \alpha > 0\) and \(\alpha_R = 0\). With the latter assumption we maintain that \(R\) is only interested in maximizing net wealth. Such an assumption is not implausible if \(R\) mainly consists of a rich elite which has the means to isolate itself from dealing with society at large.

Redistributive policies can be reduced to the choice of a wealth tax \(t \in [0, 1]\) which is levied on wealth available for redistribution. Tax revenue is evenly distributed among the population. Denoting average wealth for distribution \(\bar{w}\), utility for group \(i\) is

\[u_i(x, t) = v_i(x) + (1 - t)w_i + tw^n\]

In virtually all economies, average wealth exceeds the wealth of the median citizen so that simple electoral models predict democracy to yield majorities in favor of expropriatory taxation. Keeping the argument simple but enriching the model sufficiently to explain the empirical variety of observations we assume that taxes cause efficiency losses. Efficiency losses reduce the value of wealth which is available for redistribution, in particular if redistributive policies target capital invested in production. We assume that of \(w_R\) only a share \(\Theta_R < 1\) is available for redistribution and only a share \(\Theta_M < 1\) of \(w_M\). We assume that efficiency losses do not overturn the wealth ranking of groups when measuring wealth available for redistribution, i.e. \(\Theta_Rw_R > \Theta_Mw_M > w_L\). Thus, average wealth available for redistribution is \(\bar{w} = s_R\Theta_Rw_R + s_M\Theta_Mw_M + s_Lw_L\). We define the wealth gap of each group rel-

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7What is missing from our model are rules governing choices such as electoral rules yet as we argue below, in Chile’s case electoral rules favored conservative policies. We also ignore individual concerns of members of the regime such as illegitimate acquisition of property or human rights violations, the letter being a main issue for the Chilean Junta. Such concerns may be captured by \(R\)’s participation constraint.

8Kitschelt (1996) finds that the majority of policy choices can be subsumed under a distributional/community dimension. Schofield and Caitafe (2007) use a labor-capital dimension and hard currency-soft currency dimension in order to explain electoral positions in Argentina in the early 1990s.

9Alternatively, we may interpret our specification as a reduced form of the labor-supply model in Meltzer and Richard (1981).
ative to average available wealth as $\Delta_M = w_M - \bar{w} \leq 0$, $\Delta_L = w_L - \bar{w} < 0$ and $\Delta_R = w_R - \bar{w} > 0$.$^{10}$

3 De Novo Design of the Constitution

In the absence of a status quo constitution or if the reform process based on the autocrat’s constitution is rejected by at least one party, the constitution has to be designed from scratch. Crucially, the outcome of this process can only be predicted with some uncertainty. Ex ante, the outcome takes the form of a lottery $\ell = \{(x, t, \pi(x, t))\}$ where $x$ and $t$ take at least two distinct values with strictly positive probability $\pi(x, t)$. The associated expected utility $u_i(\ell)$ defines the default outcome $u^0_i$ for each group. We define the set $I$ of constitutions, i.e. final outcomes of the constitutional reform process, which are weakly preferred by all players to the lottery $\ell$:

**Definition 1.** $I$ is the set of outcomes which are weakly preferred by all players to de novo design of the constitution, $I = \{(x, t) | (x, t) \succeq_i \ell, i = L, M, R\}$ where $\ell$ is a lottery which assumes at least two distinct pairs $(x_1, t_1), (x_2, t_2)$, $x_1 \neq x_2$, with strictly positive probability.

To see that $I$ is non-empty, let the expected value of $x$ and $t$ from the lottery $\ell$ be $(x^0, t^0)$. From concavity of $v$ and linearity of $u$ in $t$ the set of outcomes which are acceptable for all players over entering free negotiations is non empty and contains at least the policy point where the expected values $x^0$ and $t^0$ are offered. Moreover, $I$ is a meaningful, i.e. non trivial choice set:

**Lemma 1.** $I$ is non-empty, convex and does not vanish.

*Proof.* $I$ is non-empty and does not vanish, i.e. there are points which are strictly preferred by $L$, $M$ and $R$ to their default outcomes: Because preferences represented by $u_i$ are convex, $u_i(\ell) \leq u_i(x^0, t^0)$ for all $i$. Moreover, because $u_i$ is continuous and differentiable there exists a point $(x^0, t')$, $t' < t^0$, which $M$ and $L$ prefer to the default outcome and which is strictly preferred by $R$. $I$ is convex because players’ better-sets are convex, hence the intersection, $I$, is also convex. \hfill \square

We rationalize the lottery $\ell$ by a model of conflict or bargaining in the shadow of conflict.$^{11}$ Assume each party may find itself in a position to impose or propose a

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$^{10}$From the definition it follows that in the limit, as $\Theta_M w_M \to \Theta_R w_R$ we have $-\Delta_L = w_L (1 - s_L) - (1 - s_L) \Theta_M w_M$ and $\Delta_M = w_L s_L - s_L \Theta_M w_M$ and, hence, $\Delta_M = \frac{s_L}{1 - s_L} |\Delta_L|$.

$^{11}$See e.g. Grossman (2002) for a related model of conflict.
policy point and that $\pi_j$ is the probability of this happening. Moreover, we assume that the cost of further descent into conflict or continued bargaining are such that an offer of a party’s most preferred policy point is accepted by all other parties. Thus, if $L$ wins, the policy realization $(t, x)$ is $(1, x_L)$ and if $M$ wins, the policy realization is $(1, x_M)$ for $w_M < \bar{w}$ and $(0, x_M)$ for $w_M > \bar{w}$. If $R$ wins, it selects a tax rate of zero and, being indifferent about $x$, mixes $x_M$ and $x_L$ with probabilities $\pi_M$ and $\pi_L$.\footnote{This assumption is made for convenience and considerably simplifies our formal derivations.}

For agent $i = L, M, R$, expected utility from de novo design of the constitution is

\begin{align}
\begin{aligned}
\quad u_i^0 &= \frac{\pi_M}{\pi_M + \pi_L} v_i(x_M) + \frac{\pi_L}{\pi_M + \pi_L} v_i(x_L) + (1 - \pi_L) w_i + \pi_L \bar{w} - K_i \\
\text{if } w_M &> \bar{w}, \tag{1}
\quad u_i^0 &= \frac{\pi_M}{\pi_M + \pi_L} v_i(x_M) + \frac{\pi_L}{\pi_M + \pi_L} v_i(x_L) + \pi_R w_i + (1 - \pi_R) \bar{w} - K_i \\
\text{if } w_M &< \bar{w}. \tag{2}
\end{aligned}
\end{align}

where $K_i$ is the cost incurred by party $i$ when it enters the de novo design stage. While $K_i$ may assume the value zero for some $i$, it is crucial for our results to hold that $\pi_i > 0$ for all $i$ so that the lottery $\ell$ in Lemma 1 does not degenerate.

4 Choice of a Constitutional Template $c^*$ in the Static Model

At the bargaining stage, a bargaining coalition $S$ forms of two or three players where bargaining takes the form of a player $i \in S$ proposing a reform constitution $(x, t)$ as a take-it-or-leave-it offer to the other members of the bargaining coalition with an ex-ante probability of $P_i(S)$ which is negative monotonic in coalition size, i.e. for the grand coalition $N$, $P_i(N) < P_i(S)$ for $S \subset N$ and $i \in S$. For our characterization results, we additionally impose the assumption that $P_i(\{i, j\}) = 1/2$ for all $i, j$. Note that player $k$ who is unhappy with the bargaining outcome can realize her default pay-off $u_k^0$ by invoking the conflict scenario. There are no side payments.\footnote{See our discussion in section 7. While in the dynamic spatial model of Baron, Diermeier and Fong (2012) the outcome induced by the status quo is deterministic and in the case of coalitions of two coincides with the midpoint of the contract curve, in our case the induced outcome is a lottery over points on the contract curve. In this we assume that the existence of a status quo constitution does not eliminate, but only reduces uncertainty.}

Assume that the autocrat has handed down a constitutional template $c = (x^c, t^c)$. A constitution not in the set $I$ will be rationally objected by at least one player when the status quo outcome is implemented. Therefore, we focus on status quo
constitutions handed down by the autocrat which are included in $I$. Given $I$, a status quo constitution $c$ and a bargaining coalition $\{i, j\}$ formed by two players, a rational proposal by player $i$, $(x, t)$, will maximize $u_i(x, t)$ subject to $(x, t) \in I$ and subject to the other player, $j$, realizing her reversion outcome from rejecting the reform constitution and ending up with $c$, $u_j(x^c, t^c)$. A status quo constitution $c$ is acceptable to the members of a bargaining coalition $S$ as a basis of their negotiations, if each member $i$’s expected pay-off under the bargaining protocol given $c$, $EU_{i|\{c, S\}}$ is at least as great as her default pay-off $u_0^i$.

An equilibrium consists of a status quo constitution $c$, a coalition of bargainers and bargaining moves. We say that given $c$, in equilibrium a bargaining coalition $S$ may form unless there is a coalition of bargainers of at least two players $T$ which is strictly preferred to $S$ by all its members. That is, we require the choice of bargaining coalition to form a strong Nash equilibrium.\footnote{The requirement is quite intuitive but there is no simple bargaining protocol which renders the same outcome as a Nash equilibrium. An alternative reference to the core may result in a set-valued solution. Strong Nash equilibrium and its relation to coalition formation games have been studied by Konishi, Breton and Weber (1997) and Pech (2012). See Baron and Diermeier (2001) and Baron, Diermeier and Fong (2012) for an application in a voting framework.} Accordingly, we define equilibrium for the game:

**Definition 2.** An equilibrium of the game consists of a choice for the constitutional status quo, $c^*$, a bargaining coalition of at least two players and corresponding bargaining moves such that no two or three actors strictly prefer entering negotiations with each other and individual strategies are sequentially rational.

Sequential rationality implies that a player’s decision must be part of an optimal strategy for the remainder of the game.\footnote{See Kreps and Wilson, 1982.} A strong Nash equilibrium need not exist - much in the same way that the core of this coalition formation game may be empty. Hence, it is not a priori clear that this game has a well-defined solution: Let $\{i, j\}$ denote a coalition of $i$ and $j$ and $\{i, j\} \succ_{ij} \{i, k\}$ if the expected pay off from bargaining between $i$ and $j$ is greater for $i$ and $j$ than either’s expected pay off if bargaining takes place between $i$ and $k$. As Lemma 5 below shows, the intra-coalition bargaining game is such that $\{i, j\} \succ_{ij} \{i, k\}$ for all pairs, $\{i, j\}$. Hence, strong Nash equilibrium fails to exist if (and only if) there is a sequence of the kind $\{i, k\} \succ_{ik} \{i, j\}$, $\{i, j\} \succ_{ij} \{i, k\}$ and $\{k, j\} \succ_{kj} \{k, i\}$. As we show in Lemma 2 below, with the equilibrium constitution $c^* M$ is indifferent between negotiating with $R$ and $L$. Hence not all three strict preference relations can simultaneously hold at $c^*$. 


Figure 1 depicts the bargaining situation where $M$ has more than average wealth. In the diagram, the set $I$ is the space bounded by the indifference curves corresponding to the default utility levels $l^0$ for $L$, $m^0$ for $M$ and $r^0$, which is the minimum of the tax rate corresponding to $R$’s reversion utility level, and the line $t = 1$. $MCCL$ is the contract curve of $L$ and $M$. $l^0$ is downward sloping in $x < 1$ and $m^0$ is downward sloping in $x \geq 0$. Let $p^{ij}$ be the proposal which $i$ makes as proposal maker to responder $j$ and the proposal $p^{ji}$ which $j$ makes as proposer to responder $i$. Assume that $M$ and $R$ bargain. Given $c$, $M$ proposes to $R$ the point $p^{MR}$ with the tax rate $t^c$ and her most preferred $x$ on $L$’s participation constraint $l^0$. As $R$ wants to minimize $t$, his proposal is given by the intersection of $l^0$ and $m^0$. If $M$ and $L$ were to bargain with each other, $M$ would propose to $L$ point $p^{ML}$ which maximizes $M$’s utility given $L$’s continuation utility level on $l^0$. $L$ would offer $M$ her default utility level $m^0$ and also satisfy $R$’s default utility level $r^0$.

As we show more formally in Proposition 1, the equilibrium constitution $c^*$ is located in the efficient range on the right-hand-side of the vertical part of the contract curve of $L$ and $M$, $CC$, and in the range $x \leq 1$: In this range, any move which is agreed by $L$ and $M$ is opposed by $R$. At the equilibrium constitution $c^*$, $M$ is indifferent between bargaining with $L$ and $R$:

**Lemma 2.** Assume $w_M > \bar{w}$ and the autocrat wants to design a constitution for negotiations between $R$ and $M$. In equilibrium, the autocrat must offer $M$ a constitution $c^*$ such that she is indifferent between negotiating with $R$ or $L$.

*Proof.* Assume only two-player coalitions form and suppose that given $c$, $M$ would strictly prefer negotiating with $L$ to negotiating with $R$. As $L$ also prefers being included in negotiations and $R$ suffers a loss if excluded, this cannot be in $R$’s interest: Consider in particular point $e$ in figure 1 which is $R$’s preferred point in $I$. After selecting $c^* = e$, $L$ and $M$ are predicted to bargain. In this coalition, $M$ would propose point $b$ and $L$ would propose the point in the intersection of $r^0$ and $m^0$. Both offers are worse for $R$ than points $d$ and $e$ which are offered in bargaining between $M$ and $R$. Note that by construction (see lemma 3), $b$ must always lie to the north east of $d$.

Next suppose that $M$ strictly prefers negotiating with $R$. Now $R$ would benefit by extracting a rent from $M$ because in equilibrium there is a trade-off between $M$’s pay-off $EU_{M|\{c,\{M,R\}\}}$ and $R$’s pay-off $EU_{R|\{c,\{M,R\}\}}$ as we demonstrate:

As figure 1 illustrates, if, in the range $x < 1$, $c$ moves in the direction of increased $x$ and downwards along indifference curve $m^0$, $M$’s expected utility in a coalition with $R$ decreases and $R$’s expected utility increases: Any such sequence of moves results in a move of $p^{MR}$ and $p^{RM}$ downward and right along $L$’s participation constraint.
Because $R$ prefers smaller $t$ and $M$’s indifference curves cross $l^0$ from above, the trade-off obtains, as claimed. Moreover, because the autocrat’s problem is to maximize a quasi-concave function on a convex set, any increasing sequence must lead to the overall optimum. Finally, consider the case where $I$ includes $x = 1$ and let $c$ move downwards along the vertical line $x = 1$. This move does not affect $p^{RM}$ which is selected at the minimum of $I$ but it results in a move of $p^{MR}$ downward and right, as above.

This result does not extend to the case $w_M < \bar{w}$ (see Proposition 2 and the discussion thereof). Note that although $M$ is indifferent between negotiating with $R$ and $L$, in equilibrium $M$ must settle for negotiations with $R$. Otherwise, $R$ would have wanted to deviate and design an optimal constitution for negotiations with $L$.

The following lemma provides a characterization of equilibrium for the case where $R$ designs the constitution to negotiate with above-average effective wealth $M$ and
the constraint on $t$ is not binding.\footnote{The constrained case is considered in the proof of Proposition 1.} For our characterization results, we impose the assumption that recognition probabilities in a coalition of two are symmetric, i.e. $P_i(\{i, j\}) = 1/2$ for all $i, j$. It turns out that the equilibrium constitution $c^*$ is located on $m_0^0$ if in the range $x < 1$ or on the intersection of the vertical line $x = 1$ and the betterset $B_M = \{(x, t)|u_M(x, t) \geq u_M^0\}$. We denominate this set $A_M$.\footnote{Formally, $A_M = (m_0 \cap \{(x, t)|x < 1\}) \cup (B_M \cap \{(x, t)|x = 1\})$.}

If $L$ and $M$ negotiate, the proposal is located on the contract curve CC, because the horizontal part of the contract curve is unattainable, on the part of $r^0$ between the $CC$ and $A_M$. We denominate this set $C_R$.\footnote{Formally, $C_R = \{(x, t)|(x, t) \in CC \land t \leq r^0\} \cup \{(x, t)\in (r^0 \land I) \land x \geq x_{eff}\}$ where $x_{eff}$ is the ordinate value of the $CC$ line.}

**Lemma 3.** Assume that $w_M > \overline{w}$ and the non-negativity constraint on $t$ is not binding. If $M$ is predicted to negotiate with $R$, in equilibrium $c^*$ must satisfy Lemma 2 and the following conditions: (a) $c^*$ must lie on the same indifference curve of $L$ as the induced proposal $p^{ML}$. (b) The induced proposals $p^{ML} \in C_R$ and $p^{MR}$ lie on the same indifference curve of $M$. (c) $c^*$ lies on $A_M$ and gives $M$ at least her continuation utility $u_M^0$. (d) $p^{MR}$ lies on $l^0$ and gives $L$ her continuation utility $u_L^0$.\footnote{The constrained case is considered in the proof of Proposition 1.}

**Proof.** See part 11.1 of the appendix.

Now consider the case where $M$ has less than average effective wealth:

**Lemma 4.** Assume that $w_M < \overline{w}$. In equilibrium $c^*$ must be located on $r^0$ and $M$ offers $R$ his default pay-off.

**Proof.** See part 11.2 of the appendix.

Even in the case where where $w_M < \overline{w}$, the autocrat prefers handing down a constitution in the static model as long as $R$ has positive probability of proposing in a coalition with $M$ or $L$. This contrasts to Michalak/Pech (2013) where bargaining coalitions are exogenous: There, in the case where $L$ and $M$ are predicted to negotiate a positive value fails to be created for $R$. If bargaining coalitions are endogenous and $w_M < \overline{w}$, the coalition of $L$ and $M$ never forms because under the optimal constitution $M$ (or $L$) realizes a greater pay-off when negotiating with $R$. Note that unlike in the case of Lemma 2, $R$ cannot benefit from lowering $M$’s pay-off because they realize a corner solution.

Finally, we show that the grand coalition never dominates bargaining in coalitions of two for all its members:
Figure 2: The case $w_M < \bar{w}$: With constitution $c'$, $M$ prefers bargaining with $L$. 
Lemma 5. Each player strictly prefers to be included in two player negotiations over being the out player in a two player coalition or negotiating in a three-player coalition.

Proof. See part 11.3 of the appendix.

Proposition 1 summarizes and extends our results:

**Proposition 1.** For the static model of constitutional choice there exists a constitution which the autocrat wants to hand down. Moreover, it is an equilibrium for the reform negotiations to take place between R and M and the equilibrium constitution written for this bargaining coalition is unique.

Proof. See part 11.4 of the appendix.

Note that generically the optimal constitution is not unique, i.e. there is an optimal constitution for \{L, R\} and an optimal constitution for \{M, R\}. However, in the sequential equilibrium on which we focus where R expects to negotiate with M, he writes the constitution accordingly and finds his expectation fulfilled. We can also show that the assumption for our characterization results that \(P_i(\{i, j\}) = 1/2\) for all \(i, j\) is sufficient for R to be ex ante indifferent between bargaining with L and M:

**Remark 1.** Assume the selection probabilities in negotiations with R satisfy \(P_M(\{M, R\}) = P_L(\{L, R\})\). The autocrat is indifferent between designing a constitution for M - when M and R are predicted to bargain - or for L - if L and R are predicted to bargain.

Proof. See part 11.5 of the appendix.

In this paper, as in our motivating example, we focus on the case where the autocrat designs the status quo constitution for a bargaining coalition of M and R. Note that although the autocrat is ex ante indifferent between designing a constitution for M or L, once c is designed to support negotiations with M, R strictly prefers bargaining with M rather than L unless a corner solution is realized where all bargains give R a tax of zero.

5 Efficiency of the solution

The constitution selected by the autocrat is Pareto-efficient if the middle class M is opposed to redistribution.

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Corollary to Proposition 1. Assume $w_M > \overline{w}$. In this case, the status quo constitution $c^*$ selected by the autocrat is Pareto-efficient.

Proof. The set of efficient points in $I$ is bounded by the $CC$-line and the line $x = 1$. The claims follows directly from Proposition 1 and Lemma 3.

As the corollary demonstrates, as long as the interests of the middle class $M$ and the rich $R$ over the tax rate are in harmony, the autocrat does not want to disadvantage $M$ by imposing an inefficient allocation of $x > 1$ with the status quo constitution. Proposition 2 below shows that this result does not extend to the case of conflicting interests. In this case $c$ is located in $I \cap r^0$ and $M$ and $L$ strictly prefer negotiating with $R$ rather than negotiating with each other. Hence, the autocrat would like to extract this surplus. This is possible, if the feasible set $I$ includes the point $(1, r^0)$ or $(0, r^0)$ so the autocrat may offer an inefficient point.

**Proposition 2.** Assume $w_M < \overline{w}$. If the the feasible set $I$ includes the point $(1, r^0)$ and $M$ and $R$ bargain or $(0, r^0)$ and $L$ and $R$ bargain, the autocrat selects an inefficient status quo constitution $c^*$.

Proof. See part 11.6 of the appendix.

In the case where the middle class supports redistribution and the feasible set permits, the autocrat can, in view of Lemma 2, (partially) extract any surplus above the default outcome that the other parties can enforce by bargaining with each other. He achieves this outcome by selecting a constitutional status quo point outside of the efficient domain.

Note that our results contrast to results obtained by Baron, Diermeier and Fong (2012) within their dynamic spatial legislative bargaining model. There, it is the party disadvantaged by the status quo which is selected to form a government in the second period and, therefore, the formateur/agenda setter in the initial period has an incentive to lower its pay-off. In our model, in strong Nash equilibrium the other parties can enforce their default pay-off from negotiating with each other. The autocrat/agenda setter, on the other hand, wants to extract any pay-off above the reversion pay-off. He always succeeds in doing this by selecting an efficient status quo constitution in the case where the middle class has above average effective wealth. But, if feasible, he resorts to selecting an inefficient status quo constitution in the case where the middle class has below average effective wealth.

$c$ implements a Pareto-efficient policy point but although $R$ is indifferent about $x$, it does not pick a point on the vertical part of the contract curve of $L$ and $M$, $CC$. The motive is similar as in Baron, Diermeier and Fong: There, a status quo point
outside of the Parto-field strengthens the bargaining position of a future proposal maker against the party currently left outside of the governing coalition. In our model, the autocrat wants to simultaneously weaken the bargaining position of the left and the center party in negotiations with the right and, hence, distorts the policy \( x \) by selecting a point off the vertical part of the contract curve \( CC \): Strengthening \( L \) would directly work against the interests of \( R \) while strengthening \( M \) would require a compromise on tax in order not to violate \( L \)'s participation constraint.

6 The Effect of Middle Class Wealth

Although by Proposition 1, a constitution which is (weakly) preferred to the default outcome by the bargainers as well as the out-party of a two-player coalition generally exists, the constitution offers different levels of safeguards against redistribution: From Lemma 4 we know that in the case \( w_M < \bar{w} \) the tax rate of the status quo constitution coincides with \( R \)'s default tax rate in the absence of a constitution. So at least in the case where \( M \) (or \( L \)) proposes in a bargaining coalition with \( R \), the constitution offers no additional safeguard against redistribution. As middle class wealth switches from a wealth-level slightly below to a wealth level slightly above average wealth, the default tax rate decreases whilst the pay-off for \( R \) from designing a constitution increases by a strictly positive, discrete magnitude:

**Proposition 3.** Assume that the non-negativity constraint on \( t^c \) is not binding. For \( \Delta_M \to 0 \) and \( K_i = 0 \), \( i = R, M, L \), a switch in middle class wealth from \( w_M = \bar{w}^{(-)} \) to \( w_M = \bar{w}^{(+)}, \) increases \( R \)'s pay-off from designing a constitution by a strictly positive, discrete magnitude.

*Proof.* See part 11.7 of the appendix.

Next, we provide comparative statics results for incremental increases of \( w_M \):

**Proposition 4.** Assume that the non-negativity constraint on \( t^c \) is not binding, \( w_M > \bar{w} \) and \( \pi_L = \pi_M \). For \( s_M \) and/or \( \Theta_M \) sufficiently small, \( R \)'s pay-off from designing a constitution increases as \( M \) gets richer.

*Proof.* See part 4 of the appendix.

Generally, the effect of an increase in middle class wealth on the equilibrium constitution and induced tax policies is ambiguous and depends on the size of the middle class, \( s_M \) and the effectivity \( \Theta_M \) with which middle class wealth can be taxed. An increase in \( w_M \) shifts the \( m^0 \) curve downwards in the range \( x < \sqrt{\frac{\pi_L}{\pi_L+\pi_M}} \) and the
$l^0$ curve upwards in the range $x > 1 - \sqrt{\frac{\pi_M}{\pi_L+\pi_M}}$.\footnote{Differentiating $t_{l^0}$ for fixed $x$ gives $-s_M\Theta_M \frac{(1-x)}{(w_L-w)^2}[\frac{\pi_M}{\pi_L+\pi_M} - (1-x)^2]$ and differentiating $t_{m^0}$ gives $(1 - s_M\Theta_M) \frac{(1-x)}{(w_M-w)^2}[\frac{\pi_L}{\pi_L+\pi_M} - x^2]$ where we have used $\Delta = w_i - \overline{w}$ and the definition of $\overline{w}$.} As $M$ becomes richer, she becomes relatively more averse to taxation compared to the policy goal. The countervailing effect results because as $M$ becomes richer, the value of confiscatory taxation in the conflict scenario increases and for $L$ the attractiveness of taxation increases relative to the policy goal. Hence, $L$ will demand a higher tax rate to satisfy her participation constraint. This countervailing effect is weakened when taxation is less effective in raising the income of a representative member of the left, i.e. if the effectiveness ratio $\Theta$ decreases, or if the size of the middle class, $s_M$, decreases.

To put into perspective the possibly counter-intuitive effect of middle class size, one has to bear in mind that what drives the countervailing effect is the size of the tax base and its impact on the default outcome of the left. The other determinant of this default outcome, the probability of winning conflict, is taken to be exogenous even when a larger middle class is likely to have an effect on probabilities. Yet this effect is not taken into account as we vary class size independently of $\pi_M$.

7 A Model of Intertemporal Constitutional Choice

The previous section has introduced a static model of constitutional choice where the autocrat can choose the constitution for his successor government without incurring any cost such as being bound by the constitution himself. In practice, it is likely to be a condition for a constitution to be considered as a template that it has actually been adhered to for some time before the regime’s demise.\footnote{See the discussion in the introduction.} In addition, the autocrat may not know the precise date of his demise and, therefore, will want to write and implement the constitution at a time when the probability that he will be in post for another day is still greater than zero. On the other hand, the consequences of successfully handing down a constitution might be felt for a long time. Therefore, it is reasonable to assume that the autocrat will attach non zero weights to the cost which he incurs by not realizing his preferred policy outcome $t = 0$ during the time when he has to abide by the constitution himself and to the gains which are realized at the time when his successors deliver preferred policy outcomes. We assume, that depending on the expected length of time in both states and the discount rate of the autocrat, these weights assume the values $(1 - \delta)$ and $\delta$. We take the weights to be exogenous even if $(1 - \delta)$ depends positively on his time in office which might be dependent on the constitutional choice of the autocrat. The problem of the autocrat
is to choose among the constitutions which form an equilibrium in the second stage of the game the one which gives him the highest total benefit

\[ W^R(c) = (1 - \delta)u_R(c) + \delta EU_{R\mid\{c,M,R\}} \] (3)

where \( EU_{R\mid\{c,M,R\}} \) is the expected pay-off for \( R \) if \( c \) is the constitution and the bargaining coalition is \( \{ M, R \} \) The autocrat wants to write a constitution if there exists \( c \in I \) such that

\[ W_R(c) > (1 - \delta)u_R(t = 0) + \delta u^0_R \] (4)

Prima facie it is not clear whether the autocrat wants to select the equilibrium constitution from the static problem when facing the dynamic problem: In the dynamic model, the autocrat could face a trade off between paying lower taxes today and having an optimally designed constitution after transition. However, if the autocrat lowers \( t^c \) compared to the optimal constitution \( c^* \) of the static model, it becomes more attractive for \( M \) to bargain with \( L \) rather than \( R \) thus violating Lemma 2. Hence we have:

**Proposition 5.** In the case \( w^M > w \), there is \( \delta' \) such that for \( \delta \geq \delta' \) an autocrat who wants to hand down a constitution in the dynamic setting selects the same constitution as in the static setting of Proposition 1.

**Proof.** See part 11.9

By Proposition 5, if the autocrat is sufficiently patient, i.e. for \( \delta \to 1 \), the autocrat wants to select the statically optimal constitution. Because he always wants to hand down a constitution in the static setting, this is also true of the dynamic model for \( \delta \to 1 \). On the other hand, he never wants to hand down a constitution for \( \delta \to 0 \). We cannot rule out that for small \( \delta \) the autocrat may want to hand down

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21In dynamic spatial models of government formation and policy choice the proposal maker may have incentives to distort the policy of a static equilibrium in order to decrease the demand of another party when it joins the government formed in the following period, see Baron, Diemer and Fong (2012).

22If the autocrat is motivated by an intertemporal incentive contract with his supporters (see footnote 6), it is the supporters who need to be sufficiently patient.
a constitution which is not optimal in the static model where he accepts that $M$ and $L$ negotiate the reform constitution.

It is worth stressing that in the range of $\delta$ where the optimal static constitution is also the optimal dynamic constitution, (3) and (4) represent a time-consistent policy rule: If a juncture is reached where the autocrat has to choose between the expected outcome from abiding by the constitutional process, $EU_{R\{c,S\}}$, or the default outcome from conflict, $u^0_R$, he will want to abide by the constitutional process.

8 Different modeling assumptions

In contrast to the two dimensions/three party case where weak incentives for constitution writing exist even with two parties in favour of redistribution, reducing the dimensionality of political conflict to a conflict over tax, destroys any incentives for constitution writing unless there is an exogenous cost to conflict: Assume only $L$ and $R$ are in the political arena and support by both is necessary for the constitutional reform process to succeed. Also assume, that there is a positive cost to conflict $K_L$ for the left party. In this case, the smallest acceptable tax rate for $L$, $t^{\text{min}}_L$, is less than the expected tax rate under the conflict scenario $t^0$. The autocrat now chooses $c = (t^{\text{min}}_L)$, the left party accepts the constitution $c$ and $t^{\text{min}}_L$ is realized with certainty. Because $t^{\text{min}}_L < t^0$, the autocrat realizes a surplus. If, on the other hand, $K_L = 0$, there is no advantage to writing a constitution.\(^{23}\) In the case with two dimensions and three parties, $R$ can still obtain an advantage when he is proposing against the default tax rate by making concessions on the policy dimension $x$.

We may also consider a model where players can make side payments to other political parties. Ex post an efficient policy point with $x = x^{\text{eff}}$ on the contract curve of $L$ and $M$ is realized and each proposer fulfills the participation constraint of the out-player and gives the utility with the status quo constitution to the other player in the bargaining coalition. In strong Nash equilibrium, the player with the greatest pay-off in the status quo constitution is never included in a bargaining coalition because the other two players always do better by excluding this player. So in terms of default pay-offs $R$ would want to rank as the second player. In the limit, the autocrat matches $R$’s pay-off with the status quo constitution and $L$’s pay-off with the status quo constitution and lowers $M$’s pay-off to $u^0_M$. $M$ is now indifferent between negotiating with $R$ and $L$ and, in equilibrium, negotiates with $R$.

Clearly, the assumption that there are transfers that can be realized in the constitutional reform process is rather strong. Also, the implication that the left party

\(^{23}\)Risk aversion with respect to wealth has the same effect as assuming costs of conflict.
is favored by the status quo constitution is clearly at odds with the Chilean case.

9 Reappraising the 1980 constitution of Chile

In comparing our modeling approach with the Chilean experience, we have to ask how well the model fits the choices embedded in the Chilean transition process, whether it can be thought of as a fair representation of the motivations of the main actors and makes correct predictions.

We first provide some background. The Chilean constitution of 1983 consisted of two parts: a permanent part which laid the basis of republican institutions and a transitory part which confirmed the status quo of the dictatorship. At the same time, it limited the term of the Junta and ultimately set out a transition to civilian government within a ”self-protected democracy” (Barros, 2002, p 169). It is, therefore, plausible that the Junta ultimately intended to design a constitutional order for a succeeding democratic government.

The constitution set a definite term limit on Pinochet’s presidency because in its permanent part it ruled out re-election of the president. Taking into account that Pinochet did not expect defeat in the presidential elections of 1988 (Montes/Vial, 2012) the constitution effectively limited his tenure to 16 years, a sufficiently long spell to justify our modeling assumption that any change of type of government is exogenous to the constitution itself. Whether Pinochett was driving force or reluctant follower in this process has been disputed. Yet he was probably aware of the potential role of constitution in establishing a post-transition status quo: In 1977 he had met with Hayek, who to some extent supported the idea of transitional dictatorship, and received a copy of Hayek’s ”Model Constitution” (see Farrant/McPhail/Berger, 2012).

The model idealizes but captures basic elements of the Chilean transition process. The assumption that a single autocrat designed the constitution is a simplification. The 1980 constitution was worked out by the Junta based on a draft formulated by the Constituent Commission - a committee of conservative constitutional lawyers mainly drawn from the right and complemented by some Christian Democrats (Barros, 2002) and approved in a controversial plebiscite.

After Pinochet’s electoral defeat in 1988 under the terms of the constitution, negotiations on constitutional reform were conducted between representatives of the center-left (Concertacion) and the right (RN) with the government weighing in on

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24 Barros forcefully argues in favor of the latter.
25 For an overview see Montes/Vial (2005) and Barros (2002).
some issues, parties of the left were not admitted.

Although the negotiations about constitutional reform were mainly about procedure and composition of decision making bodies, the rules adopted had a lasting impact on policies. Because the right manged to hold on to appointed senators - an arrangement only jettisoned in the constitutional reform of August 2005 - it was able to block legislation in the senate. As a consequence, the neo-liberal reforms introduced by the regime were continued. Yet even after the normalization of political institutions, the center-left continued the business-friendly policy which played out favourably with international investors (see, e.g. European Commission, 2007). This suggests that the right and the center-left were united in their opposition to redistribution and that in the transition process the right successfully defend the (constitutional) status quo in terms of redistributive policies.\textsuperscript{26} The Concertacion ended up as the dominant political force and was able to go on about day-to-day politics as long as it did not cross the red lines which the right was able to defend using the constitutional safeguards.

The most important change with the reform constitution was the rewording of article 8 which had banned Marxist parties to allow political pluralism. This enabled parties of the left which were not anti-system to run in elections. This suggests that the constitutional assembly was concerned about the acceptability of the reform constitution to a large part of the population. When the reformed constitution was put before a referendum it obtained wide-spread support, even by parts the left, which initially opposed the constitutional reform project (see e.g. Tapia, 1987). In the end, the reform proposal was supported by the unions and only the Communist Party called for a boycot of the referendum - suggesting that the participation constraint of a significant section of the left had been met.\textsuperscript{27} The overall assessment of the developments is of a reduction in political polarization (Montes/Vial) as is in line with our model.

Having laid out our case, we have to acknowledge that other authors put a different emphasis in their rationalization of the 1980 constitution. For Barros, the Junta’s need for short term stabilization prevailed over the motive of long-term liberalization. He sees the 1980 constitution as a compromise between different factions of the Junta and as partial fulfillment of a promise of non-permanency of authoritarian rule (Barros, 179 pp). In line with our approach, Montes and Vial stress the

\textsuperscript{26} The Alwyn administration immediately following Pinochet, though, increased the progressivity of the tax system - an alterantive not explicitly captured in our model.

\textsuperscript{27} We do not explicitly model the military as a player, but the presence of the military as a defender of the status quo quite likely tilted the probability of winning any conflict away from the left.
long term orientation of the constitution. They argue that given the self-image and justification of the junta, the effort at institution building can be seen under the aspect of forging a bulwark against a perceived Marxist threat.

10 Conclusion

In this paper we have characterized the constitutional status quo point which an autocrat would select if he cared about making this point acceptable to a democratically elected assembly as starting point of a constitutional reform process. We show that a sufficiently patient autocrat would want to select such a status quo point and that he would be willing to implement the statically optimal status quo constitution even if he had to accept the imposed constraints for himself.

The optimal status quo constitution in the static model is always efficient in the case where middle class and the constituency of the autocrat agree on redistribution because by choosing an efficient constitution the autocrat can extract any surplus that the other parties have above their default outcome from bargaining with each other. Only in the case of conflict can the situation arise where the autocrat wants to select an inefficient point in order to extract this surplus.

We find our modeling approach and our results largely supported by the experience of the Chilean transition process from autocracy. Our model generates comparative statics which support the idea that a well-off middle class contributes to constitutional stability - a concept which our paper makes more concrete as a measure of the willingness of an autocrat to bind himself by a constitution he wishes to hand down. A switch of effective middle class wealth from below to above average wealth has a non-marginal positive effect on the desirability of constitution writing. More generally, the relationship between the pay-off to the autocrat’s clientele and middle class wealth is ambiguous. But a positive effect can be shown if taxation is not ”too effective” in redistributing wealth.

Independently of our application, our results contribute to our understanding of the spatial voting model as we show that in equilibrium, harmony of interest between the constitutional agenda setter and the future bargaining partner of his clientele ensures that the constitutional status quo point is efficient.
11 Appendix

11.1 Proof of Lemma 3

The four conditions describe an equilibrium: We obtain \( p^{ML} \) as \( M \)'s maximum given the indifference contour \( u_L(c^*) \), so (a) is fulfilled by any rational proposal of \( M \) to \( L \). If (b) is fulfilled, the pay-off of \( M \) is equal when proposing in the coalitions \( \{M, L\} \) and \( \{M, R\} \). Because as responder \( M \) receives \( u_M(c^*) \), she receives the same pay-off as responder in the coalitions \( \{M, L\} \) and \( \{M, R\} \). So (a) and (b) fulfill Lemma 2. By (c) the constitution \( c^* \) is sustainable and by (d) \( M \)'s proposal to \( R \) is in \( I \).

To see that these conditions are also necessary, consider moving \( c^* \) downwards along \( m^0 \). Because \( p^{ML} \) fulfills (a) by construction, this results in a violation of condition (b) and \( M \) would prefer negotiating with \( M \) as \( p^{MR} \) moves down along \( l^0 \). If \( c^* \) moves up, \( R \)'s pay-off is smaller in the case where \( M \) proposes and unaffected when \( R \) proposes.

Suppose \( c^* \) moves left to \( c' \) along the iso-tax line \( t^c \) and away from \( A_M \) in violation of (c). As \( M \) enjoys greater utility at \( c' \), \( u_M(c^*) \) shifts right to \( u_M(c') \). As \( R \)'s proposal has to fulfill \( l^0 \) and offer \( u_M(c') \), \( p^{RM} \) is above and right of point \( p^{RM} \) with a higher tax. In the case where \( M \) proposes, she offers \( t^c \) as with \( c^* \).

Now consider the case where \( I \) includes the \( x = 1 \)-line and suppose that the autocrat selects \( c' = (x', t') \) and \( x' > 1 \). Note that at \( x' \) the indifference curve of \( L \), \( u_L(x', t') \), is upward sloping. By continuity there exists \( c'' \) with \( (1, t'') \) such that \( t'' < t' \), \( u_L(1, t'') \geq u_L(x', t') \), \( u_M(1, t'') > u_M(x', t') \) and the induced proposal \( p^{MR}(c') \) is at least as good for \( M \) as \( p^{MR}(c'') \). In this case, the coalition \( \{M, R\} \) may form in equilibrium and \( R \) realizes \( t'' \) with \( M \)'s proposal or the minimum tax rate in \( I \) with his own proposal.

Finally, consider \( c^* \) on the horizontal segment of \( C_R \). This fulfills all conditions, except condition (d). In this case, however, \( R \) realizes a smaller expected pay-off when negotiating with \( M \) than for the solution with \( p^{MR} \in l^0 \). Hence, conditions (a) - (d) are also necessary.

Given \( c^* \), negotiations between \( M \) and \( R \) are compatible with equilibrium: \( R \) is not better off when negotiating with \( L \): In this coalition, \( L \) proposes \( c^* \) and \( R \) proposes \( c^* \) but this is worse for \( R \) than \( p^{RM} \). \( L \) would prefer negotiating with \( M \) - where \( L \) proposes the intercept of \( m^0 \) and \( r^0 \) - but as we have shown, \( M \) is indifferent. Finally, in three-way negotiations where all agents have to agree on a proposal \( c^* \)

\(^{28}\)Note that \( c^* \) cannot coincide with point \( e \), where tax is minimized in \( I \): Suppose it did. Then \( p^{ML} \) coincides with \( c^* \) so that \( u_M(p^{ML}) \) coincides with \( m^0 \) and \( u_L(c^*) \) coincides with \( l^0 \). But because \( p^{ML} \) is on the contract curve of \( M \) and \( L \) this implies that \( m^0 \) is tangential to \( l^0 \) from which follows that \( I \) vanishes, contradicting Lemma 1.
is realized irrespective of the proposer, so \( R \) and \( M \) strictly prefer negotiating with each other.

### 11.2 Proof of Lemma 4

Suppose a constitution is selected in \( I \) but not on \( r^0 \) (for which we write \( I \setminus r^0 \)) such as \( c' \) in figure 2. \( M \) and \( R \) realize \( p^{MR'} \) when \( M \) proposes or \( p^{RM'} \) when \( R \) proposes, the latter giving \( M \) \( u_M(c') \). If \( M \) bargains with \( L \), \( L \) proposes \( p^{LM'} \) on \( r^0 \) and \( M \) proposes \( p^{ML'} \) on \( p^{ML} \). By construction, \( u_M(p^{ML'}) \geq u_M(c') \) and \( u_L(p^{ML'}) \geq u_L(c') \) and an analogous relation holds for \( p^{LM'} \) benefiting \( L \). To see that one relationship must be strict note that if \( c' \) is in the minimal point of \( I \), \( p^{ML'} \in I \setminus \{c'\} \) and \( p^{ML} \notin I \setminus \{c'\} \) dominate \( c' \) for \( M \) and \( L \). If \( c' \) is selected in any other point in \( I \), points in the set \( \{x, t| u_M(x, t) \geq u_M(c') \} \cap \{x, t| u_L(x, t) \geq u_L(c') \} \) dominate \( c' \), as illustrated in figure 2. Thus \( M \) and \( L \) prefer negotiating with each other over negotiating with \( R \). Hence, \( c' \) cannot be located in \( I \setminus r^0 \).

Point \( b \) in figure 2, in the intersection of \( r^0 \) and \( m^0 \) constitutes an equilibrium constitution for \( S = \{M, R\} \): \( M \)'s proposals in a coalition with \( R \) is \( p^{MR} \) at point \( a \) in the intersection of \( r^0 \) and \( R \) proposes \( p^{RM} \), the minimal point in \( I \) which satisfies \( L \)'s participation constraint and \( u_M(p^{RM}) \geq u_M(c) \). In a coalition of \( M \) and \( L \), point \( b \) is realized with certainty, so \( M \) realizes a smaller pay off in \( \{M, L\} \) than in \( \{M, R\} \). However, moving \( c \) to the left, \( m \) would realize a higher indifference curve and \( R \) would need to propose to \( M \) a higher tax rate than in \( p^{RM} \) which is not in \( R \)'s interest.

### 11.3 Proof of Lemma 5

a) First consider the case \( w_M > \overline{w} \): By Proposition 1, the equilibrium constitution \( c^* \) is Pareto-efficient. Hence, the grand coalition implements with certainty any constitution \( c^* \) which has been handed down. In two-player bargaining in coalition \( S \), each member of \( S \) realizes \( u(c^*) \) as a responder and a strictly better outcome than \( c^* \) as proposer. Hence, both members of \( S \) prefer \( S \) to the grand coalition. The outplayer of a two-player coalition realizes her default utility which is smaller or equal to her utility with \( c^* \).

b) Next, assume \( w_M < \overline{w} \). If \( I \) excludes the points \((0, r^0) \) and \((1, r^0) \), \( R \) chooses an efficient status quo constitution and the result follows from applying the same argument as in part a). However, as demonstrated in the proof of Proposition 2, in the case \( I \) may include \((r^0, 1) \) (or \((0, r^0) \)) and if \( R \) plans to negotiate with \( M \), he selects \( c' = (x', r^0) \) with \( x' > 1 \).

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When negotiating with $M$, $R$ realizes $r^0$ as a responder and $p^{RM}$ in the intersection of $l^0$ and the graph of $u_M(c')$ as a proposer. In the grand coalition, $R$ realizes $r^0$ with $L$ or $M$ as proposer and $p^N$ in the minimum of the graph of $u_L(c')$ as a proposer. By construction, $p^N$ is strictly inside the convex set with boundaries $l^0$, $r^0$ and the graph of $u_M(c')$. In the case where $p^N$ is not feasible, $R$ gets $t = 0$ as with $p^{RM}$. Therefore, in $N$, $R$ gets $r^0$ with probability $1 - P_R(N)$ and a worse or equal outcome than with $p^{RM}$ with probability $P_R(N)$. Because $P_R(N) < P_R(\{M, R\})$, $\{M, R\} \succ_R N$.

11.4 Proof of Proposition 1

11.4.1 Case $w_M > \overline{w}$

Lemma 3 characterizes an equilibrium solution in the case $w_M > \overline{w}$ when the non-negativity constraint on $t$ is not binding. We show that a constitution which satisfies Lemma 3 generally and uniquely exists in section 1 for the case where $I$ excludes $t = 0$ and extend this result in section 2.

1) Assume $I$ excludes $t = 0$. Let $g(t) = \{t'|u_L(x', t') = u_L(x, t) \text{ for } (x, t) \in A_M \land (x', t') \in C_R\}$ and $h(t') = \{t|u_M(x, t) = u_M(x', t') \text{ for } (x', t') \in C_R \land (x, t) \in l^0\}$. $g$ maps the tax rate corresponding to $c$ on $A_M$ into $C_R$ through $L$'s indifference curve relation. $h$ maps a point on $C_R$ back into the tax rate corresponding to point $d$ in figure 1 through $M$'s indifference curve relation. By Lemma 3, the tax rate in $d$ corresponds to the tax rate in $c$. A fixed point of the mapping $h(g(t))$ must, therefore, be the tax rate $t^e$ corresponding to $c$ such that $c$ satisfies the conditions in Lemma 3.

Denominate $t^b$ the tax rate associated with $b$ the point in the intersection of $l^0$ and $C_R$ and $t^e$ the tax rate in $e$, the point in the intersection of $A_M$ and $l^0$ where tax is minimized. Define $\overline{t} = t^b$ and $\overline{t} = t^e$.

Observe that $h(g(\overline{t})) < \overline{t}$: This follows because in $C_R$ the indifference curve corresponding to $u_M(x', t')$ cuts $u_L(x', t')$ from below and $l^0$ is decreasing in $x$. Moreover, $h(g(t)) > t$. $g$ maps $t$ into non-decreasing $t'$ corresponding to points on $C_R$. $h$ maps $t'$ corresponding to points on $C_R$ into non-increasing tax rate $t$. Thus, $h(g(t))$ is non-increasing in $t$. Hence, $h(g(t))$ has a unique fixed point.

2) The constraint $t \geq 0$ is binding.

In this case it is impossible to construct a constitution with $t^e \geq 0$, i.e. $u_L(p^*)$ cuts $m^0$ (or $m^1$) below the $t = 0$ line. Now $c^m$ is located on the $t = 0$ line, hence $R$ realizes $t = 0$ with every proposal.

Assume that $l^0$ intersects $t = 0$ on the left of $CC$: $M$'s proposals in a coalition with $L$ or with $R$, $p^{ML}$ and $p^{MR}$, are located at point $d$ in the intersection of $t = 0$ and
with \( u_M(p_{ML}) = u_M(d) \). When \( L \) proposes to \( M \) or \( R \) to \( M \) or \( L \), the proposal is \( c^* \). So for any location of \( c^* \) in \( I \) and on \( t = 0 \), \( M \) is indifferent between negotiating with \( R \) and \( L \).

Now assume that \( l^0 \) intersects \( t = 0 \) on the right of \( CC \): \( M \)'s proposal in a coalition with \( R \), \( p_{MR} \), is located at the intersection of the \( t = 0 \)-line and \( l^0 \) and \( M \)'s proposals in a coalition with \( L \), \( p_{ML} \), is on \( CC \) and satisfies \( u_L(p_{ML}) \geq u_L(c) \). Hence, \( c^* \) must be selected in the intersection of \( l^0 \) and \( t = 0 \) to ensure \( M \) is indifferent between forming a coalition with \( R \) or \( L \). Note that in both cases, \( L \) is not indifferent between negotiating with \( R \) and \( M \) but as \( R \) and \( M \) are indifferent between their choice of coalition partners, so \( \{M,R\} \) may form in strong Nash equilibrium given \( c^* \).

### 11.4.2 Case \( w_M < \overline{w} \)

By Lemma 4, \( t^c \) coincides with \( r^0 \) and \( M \) offers \( R \) his default pay-off. When \( R \) proposes, he needs to offer to \( M \) the maximum of \( u_M(c^*) \) and \( u^0_M \), so in equilibrium \( R \) selects \( c^* \) in the intersection of \( m^0 \) and \( r^0 \). By Lemma 1, \( R \) realizes a strictly better point than his default pay-off when he is the proposer.

### 11.5 Proof of remark 1

In the case where \( w_M < \overline{w} \) the claim follows immediately from the observation, that with the optimal constitution, \( R \) proposes the minimal point in \( I \) and the other party proposes a point on \( r^0 \).

In the following we show that with equal selection probability for \( L \) and \( M \), the autocrat is also ex ante indifferent between designing a constitution for \( M \) and for \( L \) in the case \( w_M > \overline{w} \):

1. Interior solution, i.e. \( p_{ML} \) is located on \( C_R \).

Designing a contract \( c^* \) for \( M \): \( c^* \in A_M \) and \( p_{ML} \) is on the contract curve \( C_R \). Policy proposals satisfy \( p_{ML} \sim_L c^* \), \( p_{MR} \in u^0_L \) and \( p_{MR} \sim_M p_{ML} \). Moreover, \( p_{MR} \sim_R c^* \).

Designing a contract \( c^{**} \) for \( L \): \( c^{**} \in l_0 \) and \( p_{LM} \) is on the extended contract curve \( C_R \). Policy proposals satisfy \( p_{LM} \sim_M c^{**} \), \( p_{LR} \sim_L p_{LM} \) and \( p_{LR} \in A_M \). Moreover, \( p_{LR} \sim_R c^* \).

It can be seen that \( c^{**} \) coincides with \( p_{MR}(c^*) \) and \( p_{LR}(c^{**}) \) coincides with \( c^* \) so that, by construction, both constitutions assign the same tax rate \( t^c \). Existence and uniqueness of \( c^{**} \) can be shown along the same lines as uniqueness of \( c^* \). By duality, \( p_{LM}(c^{**}) = p_{ML}(c^*) \).
R realizes $t^c$ when he receives a proposal from $M$ in \{\(M, R\)\} under $c^*$ and when he receives a proposal from $L$ in \{\(L, R\)\} under $c^{**}$. If $R$ proposes, he realizes the minimal tax rate in $I$ in either coalition. Hence, $R$ is indifferent between designing a constitution for $L$ and designing a constitution for $M$.

$c^{**}$ is also compatible with a strong Nash equilibrium because, given $c^{**}$, $L$ is indifferent between bargaining with $M$ and $R$: If $L$ bargains with $M$, she realizes $u_L(c^{**})$ as responder and the point $p^{LM}(c^{**})$ as proposer. If $L$ bargains with $R$, she realizes $u_L(c^{**})$ as responder and the point $p^{LR}(c^{**})$ with $c^*$, by construction $p^{LM}(c^{**}) \sim_L p^{LR}(c^{**})$.

(2) Constrained solution

In case the constraint $t \geq 0$ is binding, $p^{RL}$ is located in the intersection of $l^0$ and $t = 0$ (or in the intersection of $x = 1$ and $t = 0$ in the case of appendix part B, case 2 and $c^{**}$ is selected on $t = 0$ to support this equilibrium. As $R$ realizes $t = 0$ in every coalition, he is indifferent over the choice of bargaining coalition.

11.6 Proof of Proposition 2

We focus on the case where $R$ and $M$ are predicted to bargain and the feasible set includes $(1, r^0)$. If the feasible set $I$ includes the point $(1, r^0)$, there is a point $c' = (x', r^0)$ with $x' > 1$, $c' \in I$ such that \{\(R, M\)\} $\succ_{R,M} \{L, M\}$. With the efficient constitution $c^e = (1, r^0)$, $R$ realizes in \{\(R, M\)\} $r^0$ and either the point $p$ in the intersection of $l^0$ and the graph of $u_M(c')$ or, if infeasible, $t = 0$. With the inefficient constitution $c' = (x', r^0)$, $R$ realizes in \{\(R, M\)\} $r^0$ and the point $p'$ in the intersection of $l^0$ and the graph of $u_M(c')$. Because $u_M(c') < u_M(c^e)$, the graph of $u_M(c')$ is to the south west of the graph of $u_M(c^e)$. Hence, $R$ realizes an equal or smaller value of $t$ when bargaining under $c'$ than when bargaining under $c^e$.

11.7 Proof of Proposition 3

Setting $u_i(x, t) = u^0_i$ we obtain the indifference curve equations $m^0$, $l^0$ and $r^0$ as

$$t_{|m^0} = \frac{\alpha}{\Delta_M} \left( \frac{\pi_L}{\pi_L + \pi_M} - x^2 \right) + \pi_L + (1 - \gamma) \pi_M + \frac{K_M}{\Delta_M},$$

$$t_{|l^0} = \frac{\alpha}{\Delta_L} \left( \frac{\pi_M}{\pi_L + \pi_M} - (1 - x)^2 \right) + \pi_L + (1 - \gamma) \pi_M + \frac{K_L}{\Delta_L},$$

$$t_{|r^0} = \pi_L + (1 - \gamma) \pi_M + \frac{K_R}{\Delta_L}.$$
where $\gamma = 1$ for $w_M > \overline{w}$ and 0 else. For $K_i = 0$ the intersections of $l^0$ and $m^0$ with $t = \pi_L$ for $w_M > \overline{w}$ and $t = \pi_L + \pi_M$ for $w_M < \overline{w}$ are given by $x_1 = 1 - \frac{\pi_M}{\pi_L + \pi_M}$ and $x_2 = \sqrt{\frac{\pi_L}{\pi_L + \pi_M}}$ where $x_2 > x_1$ for $0 < \pi_i < 1$.

$r^0$ switches from $t = \pi_M + \pi_L$ to $t = \pi_L$ and $l^0$ shifts downwards by $\pi_M$ while $m^0$ approaches the vertical line $x = \sqrt{\frac{\pi_L}{\pi_L + \pi_M}}$ as $\Delta_M \to 0$. By Lemma 4, $c$ is located on $t = \pi_L + \pi_M$ for $w_M < \overline{w}$. For $w_M > \overline{w}$, $c$ must be located below $t = \pi_L$:

Let $e$ be the point in the intersection of $l^0$ and $m^0$, $f$ in the intersection of $m^0$ and $r^0$ and $g$ in the intersection of $r^0$ and $l^0$. Moreover, let $d$ denote the endogenously determined point where the line $t = t^c$ intersects with $l^0$ and let $\tilde{x}$ be the $x$-value corresponding to point $d$. Figure 3 depicts the relationship for the symmetric case $\pi_L = \pi_M$.

Assume that $p^*$ is located on the line $r^0$. Applying the conditions in Lemma 3 we get (a) $u_L(x^c, t^c) = u_L(x^*, t^c)$, (b) $u_M(\tilde{x}, t^c) = u_M(x^*, t^*)$, (c) $u_M(x^c, t^c) = u^*_M$ and (d) $u_L(\tilde{x}, t^c) = u^*_L$ with $t^* = \pi_L$.

By Lemma 1, the Euclidian distance $||e - f|| > 0$. So suppose that $||c - f|| = \epsilon$ with $\epsilon \to 0$. Construct $p^*$ in the intersection of $u_L(c')$ and $r^0$. $||p^* - f|| \to 0$ and $||d - g|| \to 0$. However, $f$ and $g$ are on different indifference curves of $M$, so $d$ and $p^*$ violate condition (b).

This shows that for $w_M > \overline{w}$, the vertical distance between $c$ and $r^0$ is strictly
positive. \(^{29}\) Because in \(\{M, R\}\), \(M\) proposes \(t^e = \pi_L\), \(R\)’s pay-off with the constitution is strictly greater than his default pay-off. On the other hand, for \(w_M < \bar{w}\), \(M\) proposes \(t^e = \pi_L + \pi_M\) by Lemma 4, guaranteeing \(R\) only his default pay-off.

So consider the proposal by \(R\) in coalition \(\{M, R\}\). If \(R\) proposes, he realizes the point \(e\) in the intersection of \(l^0\) and \(m^0\). As this point is translated by \(\pi_M\) as \(w\) switches from \(w_M \rightarrow \bar{w}(-)\) to \(w_M \rightarrow \bar{w}(+)\), \(R\)’s pay-off difference compared to his default pay-off is not affected.

### 11.8 Proof of Proposition 4

In the absence of a constitution, \(R\) realizes \(u^0_R = (1 - \pi_L)w_R + \pi_L\bar{w}\). With a constitution pay-offs are \((1 - t^e)w_R + t^e\bar{w}\) if \(M\) proposes and \((1 - t^e)w_R + t^e\bar{w}\) if \(R\) proposes. So if \(\frac{dt^e}{dw_M} < 0\) and \(\frac{dt^e}{dw_M} < 0\), \(R\)’s advantage from designing a constitution unambiguously increases in \(w_M\).

First, we demonstrate under which conditions \(\frac{dt^e}{dw_M} < 0\) holds. Differentiating the equations for \(l^0\) and \(m^0\),

\[
\begin{align*}
  u_L(x^e, t^e) &= u^0_L, \\
  u_M(x^e, t^e) &= u^0_M,
\end{align*}
\]

we obtain

\[
\begin{bmatrix}
  -2\alpha(1 - x^e) & -\Delta_L \\
  -2\alpha x^e & -\Delta_M
\end{bmatrix}
\begin{bmatrix}
  dx \\
  dt
\end{bmatrix}
= \begin{bmatrix}
  s_M\Theta_M\pi_L \\
  1 - \pi_L + s_M\Theta_M\pi_L
\end{bmatrix} dw_M
\]

Solving for \(\frac{dx}{dw_M}\) and \(\frac{dt}{dw_M}\) we obtain

\[
\begin{bmatrix}
  \frac{dx}{dw_M} \\
  \frac{dt}{dw_M}
\end{bmatrix}
= \frac{1}{D}
\begin{bmatrix}
  -\Delta_M & \Delta_L \\
  2\alpha x^e & -2\alpha(1 - x^e)
\end{bmatrix}
\begin{bmatrix}
  s_M\Theta_M\pi_L \\
  1 - \pi_L + s_M\Theta_M\pi_L
\end{bmatrix}
\]

with \(D = 2\alpha(1 - x^e)\Delta_M - 2\alpha x^e\Delta_L > 0\) for \(\Delta_M \geq 0\). This gives \(\frac{dt}{dw_M} = \frac{2\alpha}{D}[s_M\Theta_M\pi_L - (1 - x^e)(1 - \pi_L + s_M\Theta_M\pi_L)]\). It is immediate that for \(s_M\Theta_M\) sufficiently small this expression is negative.

Next, we demonstrate under which conditions \(\frac{dt^e}{dw_M} < 0\) holds: Rewriting the conditions in Lemma 3 we obtain

\[
\begin{align*}
  u_L(x^e, t^e) - u_L(x^*, t^*) &= 0 \quad (a)
\end{align*}
\]

\(^{29}\)The construction in figure 3 can be used to show that in the constrained case \(c^*\) is located at the midpoint of \(\bar{f}_e\).
\[ u_M(\hat{x}, t^c) - u_M(x^*, t^*) = 0 \quad (b) \]
\[ u_M(x^c, t^c) - u^0_M = 0 \quad (c) \]
\[ u_L(\hat{x}, t^c) - u^0_L = 0 \quad (d) \]

where we have denominated \( \hat{x} \) the \( x \)-value associated with \( p^{MR} \) at point \( d \) in figure 1. Recall that the tax associated with \( p^{MR} \) is \( t^c \). \( t^* \) and \( x^* \) are the tax rate and policy realization with \( p^* \), corresponding to \( p^{ML} \).

We need to distinguish between the cases where \( p^* \) (corresponding to \( p^{ML} \) is on the horizontal and vertical part of the contract curve of \( L \) and \( M \).

11.8.1 The constraint \( t \leq r^0 \) is not binding

In this case, \( p^* \) is located on \( CC \), the vertical part of the contract curve of \( L \) and \( M \) and \( x^* \) coincides with the efficient point \( x^{eff} \). At \( x^{eff} \), the following condition holds:

\[
\frac{dv_M}{dx} / \frac{dv_L}{dx} = \frac{\Delta_M}{\Delta_L},
\]

or

\[
\frac{x}{1 - x} = \frac{\Delta_M}{|\Delta_L|}.
\]

Implicitly differentiating we obtain

\[
\frac{x'}{x} - \frac{(1 - x)'}{1 - x} = \frac{\Delta_M'}{\Delta_M} - \frac{|\Delta_L|'}{|\Delta_L|},
\]

from which we obtain

\[
\frac{dx}{dw_M} \frac{1}{x} = \frac{1 - s_M \Theta_M}{\Delta_M} - \frac{s_M \Theta_M}{|\Delta_L|} = \beta
\]

Because \( \Delta_M < \frac{s_M}{1 - s_L} |\Delta_L| \), \( \beta \) is always positive.\(^{30}\)

Intuitively, as \( w_M \) increases, \( M \)'s indifference curve gets flatter, the efficient solution (and \( CC \)) moves right and the motive to have lower taxation gets stronger.

---

\(^{30}\)Note that the condition is \( \frac{\Delta_M}{|\Delta_L|} < \frac{1 - s_M \Theta_M}{s_L} \). We can show that if \( \frac{\Delta_M}{|\Delta_L|} < \frac{s_M}{1 - s_L} \) then the condition is fulfilled because \( \frac{1 - k}{k} < \frac{1 - ak}{ak} \).
relative to the policy goal. We treat \( x^* \) as an exogenous function of \( w_M \), hence we write \( \bar{x}^* \).

Differentiating (a) - (d) we get

\[
\begin{bmatrix}
0 & \Delta_L & 2\alpha(1-x^c) & -\Delta_L \\
-2\alpha \dot{x} & \Delta_M & 0 & -\Delta_M \\
0 & 0 & -2\alpha x^c & -\Delta_M \\
2\alpha(1-\dot{x}) & 0 & 0 & -\Delta_L
\end{bmatrix}
\begin{bmatrix}
d\dot{x} \\
dt^c \\
dx^c \\
d\bar{L}
\end{bmatrix} =
\begin{bmatrix}
2\alpha \beta (1-\bar{x}^*)\bar{x}^* + s_M \Theta_M (t^* - t^c) \\
-2\alpha \beta (\bar{x}^*)^2 - (1 - s_M \Theta_M)(t^* - t^c) \\
-2\alpha x^c \dot{x} + s_M \Theta_M (\pi_L - t^c) \\
\end{bmatrix} d\omega_M
\]

It turns out that this system of equations can be reduced to

\[
\begin{bmatrix}
2\alpha \dot{x} \Delta_L & 2\alpha(1-x^c) \Delta_M & 0 \\
0 & -2\alpha x^c & -\Delta_M \\
2\alpha(1-\dot{x}) & 0 & -\Delta_L
\end{bmatrix}
\begin{bmatrix}
d\dot{x} \\
dx^c \\
d\bar{L}
\end{bmatrix} =
\begin{bmatrix}
\Delta_M (2\alpha \beta (1-\bar{x}^*)\bar{x}^* + s_M \Theta_M (t^* - t^c)) + \Delta_L (2\alpha \beta (\bar{x}^*)^2 + (1 - s_M \Theta_M)(t^* - t^c)) \\
-2\alpha \dot{x} \Delta_L \\
4\alpha^2(1-\dot{x})x^c
\end{bmatrix} d\omega_M
\]

With the determinant \( D = 4\alpha^2 \dot{x} x^c (\Delta_L)^2 + 4\alpha^2 (1-\dot{x})(1-x^c)(\Delta_M)^2 > 0 \) we obtain the policy functions

\[
\begin{bmatrix}
\frac{d\dot{x}}{d\omega_M} \\
\frac{dx^c}{d\omega_M} \\
\frac{d\bar{L}}{d\omega_M}
\end{bmatrix} = \frac{1}{D} \begin{bmatrix}
2\alpha x^c \Delta_L & 2\alpha(1-x^c) \Delta_M \Delta_L & -2\alpha(1-x^c)(\Delta_M)^2 \\
-2\alpha(1-\dot{x}) \Delta_M & -2\alpha \dot{x} (\Delta_L)^2 & 2\alpha \dot{x} \Delta_L \Delta_M \\
4\alpha^2(1-\dot{x}) x^c & 4\alpha^2 (1-\dot{x})(1-x^c) \Delta_M & -4\alpha^2 x^c \dot{x} \Delta_L
\end{bmatrix} v
\]

where

\[
v = \begin{bmatrix}
\Delta_M (2\alpha \beta (1-\bar{x}^*)\bar{x}^* + s_M \Theta_M (t^* - t^c)) + \Delta_L (2\alpha \beta (\bar{x}^*)^2 + (1 - s_M \Theta_M)(t^* - t^c)) \\
-2\alpha \dot{x} \Delta_L \\
4\alpha^2(1-\dot{x})x^c
\end{bmatrix}
\]

Using \( \frac{\bar{r}}{1-\bar{x}^*} = \frac{\Delta_M}{\Delta_L} \), \( v \) simplifies to

30
\[ v = \left( s_M \Theta_M \Delta_M + (1 - s_M \Theta_M) \Delta_L \right) (\pi - t^c) \]

Using \( \pi_L \geq t^c \) and \( t^* \geq t^c \) and recalling \( \Delta_L < 0 \), we get \( \frac{dt^c}{dw_M} < 0 \) for \( s_M \Theta_M \) sufficiently small.

### 11.8.2 The constraint \( t \leq r^0 \) is binding.

In this case, \( p^* \) is located on \( r^0 \), i.e. \( t^* = \pi_L \) and \( x^* \) adjusts along \( r^0 \).

\[
\begin{bmatrix}
0 & -2\alpha(1 - x^*) & 2\alpha(1 - x^*) & -\Delta_L \\
-2\alpha \hat{x} & 2\alpha x^* & 0 & -\Delta_M \\
0 & 0 & -2\alpha x^* & -\Delta_M \\
2\alpha(1 - \hat{x}) & 0 & 0 & -\Delta_L
\end{bmatrix}
\begin{bmatrix}
d\hat{x} \\
dx^* \\
dx^c \\
dt^c
\end{bmatrix} =
\begin{bmatrix}
s_M \Theta_M (t^* - t^c) \\
-(1 - s_M \Theta_M)(t^* - t^c) \\
-(1 - s_M \Theta_M)(\pi_L - t^c) \\
s_M \Theta_M(\pi_L - t^c)
\end{bmatrix}
\]

reduce to

\[
\begin{bmatrix}
0 & -4\alpha \hat{x}(1 - x^*) & 4\alpha^2 x^*(1 - x^*) & -2\alpha [x^* \Delta_L + (1 - x^*) \Delta_M] \\
-4\alpha \hat{x} & 4\alpha^2 x^*(1 - x^*) & 0 & -\Delta_M \\
0 & 0 & -2\alpha x^* & -\Delta_M \\
2\alpha(1 - \hat{x}) & 0 & 0 & -\Delta_L
\end{bmatrix}
\begin{bmatrix}
d\hat{x} \\
dx^* \\
dx^c \\
dt^c
\end{bmatrix} =
\begin{bmatrix}
s_M \Theta_M (t^* - t^c) \\
-(1 - s_M \Theta_M)(t^* - t^c) \\
-(1 - s_M \Theta_M)(\pi_L - t^c) \\
s_M \Theta_M(\pi_L - t^c)
\end{bmatrix}
\]

We have \( D = -8\alpha^3 \hat{x}(1 - x^*)x^c \Delta_L - 8\alpha^3(1 - \hat{x})x^*(1 - x^*) \Delta_M - 8\alpha^3[x^* \Delta_L + (1 - x^*) \Delta_M] \).

Observe that as the constraint is binding, \( x^* \) must be on \( r^0 \cap I \) and, using \( \pi_L = \pi_M \), \( x^* \geq 1 - \sqrt{\frac{1}{2}} \) whilst the constitution \( c \) must be situated on the right of the intercept of \( m^0 \) and, hence, \( x^c \geq \sqrt{\frac{1}{2}} \).

From the efficiency condition, we know that \( \frac{x^\text{eff}}{1 - x^\text{eff}} = \frac{\Delta_M}{|\Delta_L|} \).

Hence, \( \hat{x} > x^\text{eff} \) and \( \frac{x}{1 - x} > \frac{\Delta_M}{|\Delta_L|} \).

So the first two terms in the expression of \( D \) yield a positive number:
\[
\hat{x}(1 - x^*)x^c|\Delta_L| > x^{eff}\sqrt{\frac{1}{2}(1 - \sqrt{\frac{1}{2}})}|\Delta_L|
\]
\[
= (1 - x^{eff})(1 - \sqrt{\frac{1}{2}})\sqrt{\frac{1}{2}\Delta_L|\frac{x^{eff}}{1 - x^{eff}}}
\]
\[
\geq (1 - \hat{x})x^*(1 - x^c)\Delta_M
\]
and the third term is positive as
\[
|\Delta_L|x^*
\]
\[
> x^{eff}|\Delta_L|
\]
\[
\geq x^{eff}|\Delta_L|\frac{1 - x^*}{1 - x^{eff}}
\]
\[
= \Delta_M(1 - x^*)
\]
where we have used \(x^* \geq x^{eff}\) and \(\frac{x^{eff}}{1 - x^{eff}} = \frac{\Delta_M}{|\Delta_L|}\).

With \(D > 0\), the sign of \(\frac{dt}{dw_M}\) is negative for sufficiently small \(s_M\Theta_M\).

### 11.9 Proof of Proposition 5

Obviously, in period 1 the autocrat cannot gain by selecting a higher tax rate but he might want to choose a constitution \(c^*\) which gives him a lower tax rate.

First, consider a marginal move of \(c^*\) downwards along \(m^0\).

If the constraint \(t^c \geq 0\) is binding, \(t^c\) cannot be reduced compared to the static case. In the unconstrained case, because \(p^{ML}\) fulfills condition (a) in Lemma 3 by construction, moving \(c^*\) results in a violation of condition (b) of Lemma 3 and \(M\) would prefer negotiating with \(L\) as \(p^{MR}\) moves down along \(l^0\), thus violating Lemma 2. As Lemma 5 shows, it does not pay in the static model to be the out player of a two player coalition, hence \(EU_R(\{L, M\}) < EU_R(\{M, R\})\). Because moving \(c^*\) by a marginal amount reduces the tax in the first period only marginally but loses \(R\) membership of the second period bargaining coalition, such a move never pays.

Yet it might pay for \(R\) to accept a bargaining coalition of \(L\) and \(M\) if he can reduce the tax in the first period by a non-marginal magnitude. Consider the point \(c''\) in the minimum of \(I\) (i.e., point \(e\) in figure 1). In this case, \(R\) realizes \(EU_R(\{L, M\}, c'') < EU_R(\{M, R\}, c^*)\) in the second period but \(u_R(c'') > u_R(c^*)\) in the first period as \(t'' < t^*\). From this relationship, the claim trivially follows.
11.10 Additional Appendix

This appendix is not intended for publication but provides some derivatives for part 11.8 of the appendix to facilitate the work of the referees.

\[ \frac{u_L}{dt} = -\Delta_L > 0 \forall t \]
\[ \frac{u_L}{dx} = 2\alpha(1 - x) \]
\[ \frac{u_L(x^*, t)}{dw_M} = 2\alpha\beta(1 - x^*)x^* + s_M \Theta_M t \]
\[ \frac{u_M(x, t)}{dw_M} = s_M \Theta_M t \]
\[ \frac{u_M}{dt} = -\Delta_M < 0 \]
\[ \frac{u_M}{dx} = -2\alpha x \]
\[ \frac{u_M(t, x^*)}{dw_M} = -2\alpha\beta(x^*)^2 + 1 - t(1 - s_M \Theta_M) \]
\[ \frac{u_M(x, t)}{dw_M} = 1 - t(1 - s_M \Theta_M) \]
\[ \frac{du_L^0}{dw_M} = \pi_L(s_M \Theta_M) \]
\[ \frac{du_M^0}{dw_M} = 1 - \pi_L(1 - s_M \Theta_M) \]

References


