Deficits, Coalition Governments and Budget Institutions

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ABSTRACT

We develop an intertemporal model of legislative bargaining which compares outcomes with commitment to a debt target and outcomes when the finance mix can be renegotiated. Bargaining institutions include legislative bargaining with a proposal maker, decentralised budget proposals with and without budget co-ordination and a vote buying model with universalistic features. Agreement on a balanced budget rule is only achieved under universalism. In the budget co-ordination model where expenditure is group efficient for the governing coalition there is conflict over the debt target and agreement on an effective target is only possible if a reversion debt level is exogenously imposed.

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I. Introduction

There are several striking facts about the performance of coalition governments in stabilizing the budget and the fiscal institutions under which they operate. Alesina and Perotti (1995) found that coalition governments are often unsuccessful in their attempts to commit themselves to stabilization policies and in particular that they have a high defection rate. This finding indicates intra-coalition conflicts or a time consistency problem in the absence of a commitment technology. The experience of many European Union countries in the run up to the Monetary Union suggests that governments only manage to submit to a debt target under outside pressure. Once signed up, open or concealed defection remains a likely outcome (see von Hagen, 2004). Debt targets are often subject to intense bargaining within coalition governments. One example is the case of the German 1994 coalition government where the Liberal Party (FDP) had previously made a strong commitment to a balanced budget in its party platform but abandoned that target in subsequent budget negotiations. On the other hand, the US congress - which is characterized by unstable legislative coalitions (see e.g. Baron, 1989, Persson, Roland and Tabellini, 2000) - always had a strong preference for a balanced budget. In 1995 a constitutional balanced budget amendment passed the House with more than the necessary two-thirds majority and was defeated in the Senate by only one vote. Still there are ample examples of ordinary legislation with the objective of bringing about a balanced budget. The Gramm-Rudman-Holling (GRH) legislation of 1985 and 1987 introduced a fixed deficit target with an automatic sequestration procedure. The Budget Enforcement Act of 1990 and the 1993 Omnibus Budget Reconciliation Act ruled that tax and direct spending legislation had to be "deficit neutral", i.e. every expenditure law had to be accompanied by a proposal to finance the program. It is evident that such a procedure is equivalent to the setting of a debt target. In fact, the US was able to effectively cope with the
deficit problem in the 1990’s even if it had failed to do so under the GRH legislation. If anything, the persistent attempts to tackle the procedural problems of budgetary rules can be told as a story of target setting and subsequent defection. Thurber (1997) and Doyle (1996) report incentives for non compliant behavior with legislation which is adopted late in the budget year and with effects spilling over to the next year. GRH had to be adjusted only 2 years after its imposition while a similar effort in Belgium, the plan de convergence, had to be renegotiated within one year of adoption.

The empirical literature suggests that constraints like debt targets are effective in curbing high deficits in high debt countries (see Alesina et al, 1999). The importance of budget institutions in general is emphasized by von Hagen (1992) who had constructed an index of budget institutions which covered the structure of negotiations within the government, the structure of the parliamentary process, informativeness of the budget, the flexibility of budget execution and long-term planning constraints. The first item includes a measure for the agenda setting in budget negotiations. Variants of this index have been tested successfully in a number of studies indicating that a fragmentation of the budget process results in higher deficits and expenditures (for an overview see Perotti, 1998).

The interaction between institutional design and debt policies is still little appreciated in the theoretical literature. There is, however, an increasing literature on the impact of institutional features on expenditure policies. The latter mainly addresses bargaining in the legislature and analyzes features such as the feasibility of amendments to a bill (Baron and Ferejohn 1989), the process governing the formation of coalitions (Baron 1989, Persson, Roland and Tabellini, 2000, Chari and Cole, 1995) and different scenarios for the timing of decisions (Ferejohn and Krehbiel,
Theoretical explanations of debt policies focus on broad strategic dilemmata. They identify conflicting goals between policy makers in different periods (Persson and Svensson, 1989, Alesina and Tabellini, 1990) or between interest groups in the government of one period (Velasco, 2000). In Balassone and Giordiani (2001), explicit reference is made to a bargaining framework for government decisions. However, because this is not presented in terms of an extensive game the approach does not lend itself to the analysis of the strategic role of debt in the budget process. An important part of the literature focuses on a complementary question: under what circumstance would agents agree on a debt target (Spolaore, 1993, Velasco, 1998) when they otherwise end up with an unsustainable policy. But while in many countries efforts at fiscal consolidation have been concentrated at one particular moment in time it is equally important to understand the institutional features which are capable of producing reasonable decisions on a yearly basis.

In what follows I set up a simple two period framework which extends the standard public choice model of interest group behavior (see for example, Persson, 1998) by representing the bargaining situation in each period as an extensive game. The main result is that when a decision has to be reached on a debt target before entering budget negotiations (a TA regime in our terms) outcomes are supported which are different from outcomes when financing is negotiated after or simultaneously with the expenditure decision (RE).\(^1\) Basically, targeting forces agents to internalize externalities due to the common pool property of public debt (see Velasco, 2000). If bargaining power over expenditures differs, debt targeting has a distributional component and gives rise to conflict. If there is conflict over the debt target, setting the target is only effective if a reversion debt level - other then the renegotiation level - is threatened for the case where no agreement is reached. This may explain why outside pressure has been an important factor for
many countries to agree on stabilization policies. However, where stabilization is effective, agents have an incentive to renegotiate on the target.

This paper compares the two procedures governing debt policies under different bargaining institutions which apply to government decision making. I distinguish negotiations within the cabinet, the parliament, and the congress which combine stylized features of a variety of budgetary institutions across countries. The parliamentary model gives most power to the player who is selected as a proposal maker and leaves the other members of the winning coalition - the responding parties - without proposal rights. This can be taken to be descriptive of strongly centralized budget institutions as they apply in France or in the Netherlands. In this setting, the uncertain composition of the second term coalition induces everybody to prefer positive debt in the first term. A debt target does not change the bargaining position of a member of the coalition and is, therefore, ineffective.

The cabinet model explicitly allows for proposal rights of all coalition parties. This captures features of decentralized budget procedures as they apply in Belgium or Italy. In such a system, the budget coordinator collects the bids of the other ministers. The right to put forward an initial bid matters to the extent that the budget coordinator has to find the support of her coalition partners for any amendment of the initial budget proposal. Agreement over an effective debt target is possible if the budget procedure itself yields inefficient results for the governing coalition. Ex ante, agents foresee the problems arising from the common pool property of debt and are able to move to a Pareto better position. On the other hand, if expenditure policies are efficient for the governing coalition as a whole, the only consequence of a debt target as opposed
to debt renegotiation is to alter the bargaining positions of the different coalition parties in the subsequent budget negotiations. In that case, there must be conflict over the debt target.

Finally, we compare our results to a scenario modelled on the US congress. There, expenditure decisions are mainly taken by decentralized committees and are believed to result in universalistic outcomes. A vote buying model which is based on Chari and Cole (1995) accounts for these observations. In this variant, all legislators vote for a materially balanced budget when deciding on a debt target. However, we show that the same legislators vote for positive debt if the target is renegotiated. In the latter case, the result is driven by the legislator’s awareness of the common pool problem which they face in the subsequent term.

I proceed as follows. Section 2 sets out the economic model. The rules governing expenditure policies are introduced in section 3. Section 4 presents the dynamic framework. Section 5 analyzes the bargaining game in parliament and section 6 focuses on bargaining in cabinet. Section 6A analyzes the case of fragmented decision making and 6B accounts of budget coordination. Section 7 derives the results for a framework with vote buying. Section 8 concludes.
II. The basic economic model

I consider a small open economy with fixed world market prices and a zero interest rate which is inhabited by an odd number, \( n \), of interest groups. Those groups have equal size and equal representation in the parliament. I assume that political agents are perfect delegates of the groups which they represent. Each group is represented by one agent \( i \). The reduced form of her objective function\(^2\) as seen from period \( s \) is

\[
W_i^s = E_s \sum_{t=2}^{t=s} [v(g_t^i) - f(\tau_t)] s = 1,2.
\]  

\( E_s \) is the expectations operator taken in period \( s \). All variables are given in per head values. A lower index \( h_x(x), x \neq s, t \) indicates a partial derivative; \( s \) and \( t \) indicate time periods. The objective function of group \( i \) is concave in the government’s expenditure directed to this group, \( g_t^i \), with \( v(0) = 0 \) and \( \lim v(g(0)) \to \infty \). This means that group \( i \) is interested exclusively in \( g^i \) and government expenditures have a pork barrel character as in Shepsle, Weingast and Johnson (1981). Tax payments \( \tau_t \) are general across groups, might be negative and elicit behavioral responses which cause an excess burden. So marginal cost \( f_\tau \) exceeds the payment: \( f_\tau(\tau) > 1 \) for \( \tau > 0 \) and \( f_\tau < -1 \) for \( \tau < 0 \) and \( f_\tau(0) = 1 \). The excess burden increases with tax payments or transfers, \( f_{\tau\tau} > 0 \), so public debt has real effects (see Barro, 1979). Furthermore, \( f_{\tau\tau\tau} = 0 \), which is fulfilled if the expression for the tax burden is quadratic, as is frequently assumed in the public finance literature. In each period, the government decides on current expenditures for each group \( k, g_t^k \), taxes \( \tau_t \) and debt, \( b_t \). The government budget constraint is

\[
\tau_t = b_{t-1} - b_t + G_t, t = 1, 2
\]
where we assume \( b_0 \equiv b_2 \equiv 0 \) and \( G_t = \sum_{k=1}^{k=n} g_t^k \) denotes aggregate expenditures. In order to prevent problems arising from unbounded returns, we require that \( \lim_{g_1 \to \infty} v(g_1) - f(g_1 - b) - f(b) = -\infty \) for all \( b \) and \( \lim_{g_1 \to \infty} v(g_2) - f(g_2 + b) = -\infty \) for all \( b \neq -\infty \). Preferences are common knowledge. Such an assumption seems to be relatively innocuous in the political sphere where the procedure of aggregating individual preferences into party preferences evolves in the public domain.

III. Expenditure Policies

In this section I outline, how an expenditure policy is selected for a fixed debt policy \( b \). Expenditures are decided either under decentralized proposal rights or under a coordination process. Under decentralized proposal rights, each legislator who is established with the right to propose the level of her favorite program, does so under the assumption, that each other legislators with proposal rights chooses her preferred outcome. The resulting budget proposal is either final, i.e. the cabinet proposal is waived through parliament, or voting in parliament has a more substantial role, which we analyze in detail in the vote buying scenario.

With a budget coordination process, in each period there is one agent, \( \omega \), who is selected as a budget coordinator. This agent proposes an expenditure policy, i.e. vector \( \{g_t^k\} \) which has to be adopted by a majority of parties in parliament or by the cabinet. I assume that a simple closed rule applies, so \( \omega \) makes a take it or leave it offer. Let \( M_\omega \) be the set of minimum winning coalitions including \( \omega \) and \( C \) a typical element of \( C \) consisting of \( m=(n+1)/2 \) members. The
proposal is accepted, if \( \omega \) gives to \( m-1 \) other agents their reservation pay off, \( V'_r \) associated with a reversion allocation \( \{g_r^k \}_{k=1}^{K=n} \). The reservation pay off depends on the institutional set up \( \psi \) and on the pre-selected debt level \( b \). If the proposal maker can choose \( C \) freely, her problem is to select

\[
\max_{g'_j, j \in C, C \subseteq M_\omega} W'_r((g'_j), \bar{b}) \text{ s.t. } W'_j((g'_j), \bar{b}) \geq V'_j(\bar{b}, \psi), \ j \in C \setminus \omega.
\]  

In the cabinet model, we can treat \( C \) as fixed. The unique equilibrium for \( \omega \) is to give the reservation value to \( m-1 \) agents in \( C \setminus \omega \), nothing to the other agents and claim the residual for herself. We assume that a lower bound to \( V'_r \) is \( W(\{0\}, \bar{b}) \) in the second period and \( W(\{0\}, 0) \) in the first period.\(^4\) The allocation is efficient for the coalition as a whole with first order conditions

\[
v'_r(g'_j) = (1 + \lambda_i) f_r(\tau_i, \bar{b}), \quad (4)
\]

\[
v'_j(g'_j) = \left[ \frac{m-1}{\lambda_i} + (m-1) \right] f_r(\tau_i, \bar{b}), \ j \in C \setminus \omega. \quad (5)
\]

\( \lambda_i \) is the budgetary cost for \( \omega \) of raising the utility of the representative responder by one unit. Stated otherwise, \( \lambda_i \) is the rate at which government receipts have to be devoted to the compensation of the responders if \( \omega \) increases her consumption of government income by one unit. It is easy to see that \( \lambda_i \in \left[ \lambda, m - 1 \right] \) where \( \lambda_i = m-1 \) corresponds to the equal share allocation among members of \( C \) and \( \lambda > 0 \) corresponds to the lower bound on pay offs, \( W(\{0\}, .) \) for the responders. From (4) and (5) it is obvious, that expenditures depend on the debt policy \( \bar{b} \).

With \( b = 0 \), the setting is identical in both periods, so the expenditure policy is stationary.
IV. The dynamic setting

In this section we set the budget game into a dynamic perspective. The budget game is solved backwards, determining the equilibrium in the second period first. Inserting the values for the expenditure policies \( g_2^{\psi \sigma} (b) \) given \( b \) as they result under institution \( \psi^\sigma \) into the truncated objective function (1) for period \( s=2 \) gives the value function for a player occupying position \( \pi \),

\[
V_2^\pi(b | \psi^\sigma) = v(g_2^{\psi \sigma}(b) - f(G_2^{\psi \sigma} + b)). \tag{6}
\]

Taking expectations over all possible positions yields \( E_1V_2(b | \psi^\sigma) \). Using \( E_1V_2 \), we can represent the marginal cost of debt finance in the second period as seen from period one as the sum of expected pay off differentials which a players realizes in the second period in any position weighted with the probability of being in the respective position. We assume that as of period one, agents face an equal probability of occupying any position. Differentiating \( E_1V_2 \) with respect to \( b \) one gets

\[
\mu(b | \psi^\sigma) = -\frac{1}{n} \left( (1 + \lambda_2) f_\tau(G_2 + b) - \lambda_2 \frac{\partial V_2^\pi(b)}{\partial b} \right) + \frac{m-1}{n} \frac{\partial V_2^\pi(b)}{\partial b} + \frac{m-1}{n} f_\tau(G_2 + b) \frac{d \tau}{db}. \tag{7}
\]

Debt crowds out expenditures and incurs higher taxes. The terms in the bracket is the effect on the pay off of the second period budget coordinator. It is obtained by differentiating the Lagrangian of (3), exploiting the first order condition (4) for \( \omega_\tau \). The second term refers to the change in the pay off for a typical junior coalition member \( j \) and the final term stands for the utility change of the non coalition member. We define two decision sequences for period one:
**Definition 1** Under a RE regime, \( \{g\} \) is adopted at stage one and \((b,\tau)\) is adopted at stage two. Under a TA policy, \(b\) is adopted at stage one and \((\{g\},\tau)\) at stage 2.

Figure 1 gives an overview of the timing of events under the different institutions. A RE policy takes the present expenditures as fixed and, therefore, compares the marginal cost of tax finance, \(f\), and debt finance, \(-\mu\). When selecting a debt target under TA, one has to take into account that both current period and future expenditure policies depend on the debt target selected. We denominate the current pay off

\[
R_i(b) = v(\xi_i(b) - f(\Sigma g_1(b)) - b). \tag{8}
\]

Thus, the objective function at the debt determination stage is

\[
V_i(b) = R_i(b) + E_1V_2(b). \tag{9}
\]

In part A of the appendix we show that \(\mu'(b)<0\) \(R_{bb}<0\).

V. Bargaining in Parliament

In parliament, a budget is adopted if the selected budget coordinator finds a minimum winning coalition to accept that proposal. There are many possible arrangements for the recognition process as outlined in Baron (1989). I focus on the case where the parliament acts with coalitional discipline, in which case the winning coalition \(C\) is selected by the proposal maker for the whole legislative term.

We assume that when proposing expenditure policies, the parliamentary budget coordinator \(\omega\) presents legislators with the alternative of having no government expenditures at all, i.e. the
continuation value for an agent $i$ of rejecting a proposal in the second period is $V^{\pi_i} = W^i([0], \bar{b})$.

For a legislator in the majority coalition of the second period, $v(g) - f(G + b) = v(0) - f(b)$ must be fulfilled. Therefore, the effect of increasing debt on her pay off is the effect on her reservation outcome, i.e. $\frac{\partial V^{\pi_i}(0, b_{-i})}{\partial b_{-i}} = -f_\pi(b)$ and the marginal cost of transferring debt into the future is

$$\mu(b | \psi) = -\frac{1}{n} (1 + \lambda_2) f_\pi(G_2 + b) - \hat{\lambda}_2 f_\pi(b) - m - 1 \frac{d \pi}{db} f_\pi(G_2 + b) \quad (10)$$

with $\frac{d \pi}{db} \leq 1$ and an analogous interpretation to (7).

We can now compare debt policies under the different debt policy rules. First we consider the case of parliament acting under $RE$, so we let $\psi = PRE$. Under $RE$, all legislators have the same preferences on debt when moving to the debt setting stage. Due to uncertain participation in the next period everybody prefers to restrain the future government, i.e. $b^{PRE} > 0$: Under $RE$ debt preferred by all legislators satisfies $f_\pi(G^{PRE} - b^{PRE}) = \mu(b | \psi^{PRE})$. Using $f_\pi(0) = 1$ in (10) it is straightforward to show that $b^{PRE} > 0$ as long as $\frac{d \pi}{db} < 1$.

Now consider debt targeting. Here it is important to know what happens to the reversion budget. It is plausible, that if the reversion budget has zero expenditure, zero debt is issued to finance the reversion budget and the debt target, therefore, is not binding. So the proposal maker proposes $(g^{\omega}, g^{\omega^\omega}, b^{PTA})$ against the reversion budget $(0, 0, 0)$. It is easy to see that in this case debt targeting is ineffective because it does not affect the reversion budget. Its only effect on the behavior of $\omega$ occurs through an intertemporal distortion with the actually implemented budget.

Note that this budget is optimally financed at $b^{RE}$, so any target $b^{TA} \neq b^{RE}$ would induce an
intertemporal distortion without increasing the utility of the responding legislator. Therefore, the best a responder can do in the target setting stage is to accept $b^{TA}=b^{RE}$:

**Proposition 1** If the reversion budget in parliament is the zero expenditure vector, the debt target is identical to realized debt under renegotiation.

This result shows that a debt target is effective only if it has an impact on the position of a legislator in the budget negotiations of period 1. In the next section we show that this is the case in cabinet where the impact of the debt target can be broken down into an efficiency enhancing effect - a lower deficit discourages inefficiently high expenditures in the reversion budget - and a distributional effect in the conflict between proposal maker and ordinary legislator.

**VI. Cabinet decision-making**

In the cabinet, not only the budget coordinator but each coalition member has well-defined proposal rights. Each group in government sends one spending minister who makes a proposal for the supply of the good her clientele prefers. Hereby it is implicitly assumed that at a previous government formation stage a proposal which establishes a cabinet government has been adopted and that this proposal results in the described distribution of spending ministries. As a result, in each legislative period, the government consists of a minimum winning coalition. In the following section I assume that the budget bill of the cabinet always meets a majority at the parliamentary stage, which is not explicitly modelled: No coalition party tries to improve its outcome by attempting a bargain in the parliament. This assumption can be justified because the cabinet stage serves to pre-coordinate the preferences of the parties involved (see Huber, 1996).
I discuss two different bargaining procedures:

(a) With decentralized expenditure policies, each cabinet minister proposes a budget for her own jurisdiction and the resulting vector of proposals is the overall budget. This bargaining procedure corresponds to decentralized decision making. Debt crowds in present and crowds out future expenditures, so debt policies may serve to reach a Pareto better conjectural variations equilibrium for the coalition.

b) With budget coordination, the vector of decentralized proposals is the budget proposal on the floor but one of the spending ministers, who is selected as the budget coordinator, submits a counter proposal. This coordinated budget is adopted if it receives unanimous support of all members of the government coalition. The budget coordinator serves as a residual claimant in the budget game and has an incentive to submit an efficient proposal. The unanimity rule is broadly in accordance with explicit or implicit budget rules where the executive power plays a dominant role in the budget process (see for example Wildavsky, 1988). Even with budget coordination, there remains a strategic role for debt. When accepting a proposal on a debt target, the junior coalition member has to balance a possible dynamically distorting effect with the ultimately prevailing tax/spending policy and the effect of debt on her threat point in the budget game.
A. The role for debt with decentralized expenditure policies

We think of the un-coordinated, decentralized budget proposals as resulting from Nash-behavior of the spending ministers. Equilibrium values of variables are indicated by the superscript D. For a given debt policy, $\bar{b}$ and given policies of the other ministers, each spending minister selects $g_i^{ID}$ as to satisfy

$$g_i^{ID} = \arg \max W_i(g_i^j, \{g_i^{KD}\}, \bar{b}), i = 1, \ldots, m$$

(11)

The solution is a Nash equilibrium where each $g_i^{ID}$ satisfies

$$v_g(g_i^{ID}) = f_i(g_i^{ID} + \sum_{k \in M \setminus i} g_i^{KD} + (-1)^i \bar{b})$$

(12)

The reaction functions are $\gamma_i^{ID} : \frac{dg_i^{ID}}{db} = (-1)^i \frac{f_i}{v_{gg} - mf_i^{\tau}}$ with debt crowding in current expenditures, $\gamma_i^{ID} \in \left[0, \frac{1}{m}\right]$ and crowding out future expenditures, $\gamma_2^{ID} \in \left[-\frac{1}{m}, 0\right]$. The marginal cost of debt finance, $\mu(b|\psi^D)$, is given as

$$\mu(b|\psi^D) = \frac{m}{n} V_b^{ID} - \frac{m - 1}{n} f_i(G_i^D + b) \frac{d\tau}{db}.$$  

(13)

where $V_b^{ID} = v(g_i^j)(\gamma - m \gamma - 1)$ is the expected marginal loss for a coalition member in period 2 and $f_i \frac{d\tau}{db}$ is the expected consumption loss for a non coalition member. $\frac{m}{n}$ and $\frac{m - 1}{n}$ are the probabilities of belonging to those groups. Using (12) and the reaction function, one finds the closed form (see part B of the appendix):

$$\mu = -\left(1 + \left(m - \frac{m}{n}\right)\gamma_2^{ID} v(g_i^D)\right).$$
It is easy to see that the expression in parentheses ranges from 0 to 1. $\mu$ is smaller than the future expenditure benefits foregone by a typical coalition member, $v_g(g_2)$. His is partly due to uncertain participation (i.e. $m<n$). Partly it is due to crowding out (i.e. $\gamma D<0$): with Nash behavior in the following period, issuing debt reduces public overspending: A reduction of the $m$ projects in the next period by one saves $m$ units of taxes and reduces expected benefits only by $m/n$. This latter aspect would prevail, even if all groups were involved in the future government.

With debt renegotiation, all ministers are in an equal position when moving to the next stage so each minister has the same interest with respect to debt policies. Therefore, any minister might be given the right to propose a financing scheme. The pay-off maximizing debt-level $b^{DRE}$ also maximizes the pay off for any other member of the government, so the proposal is unanimously accepted.

The preferred debt policy $b^{DRE}$ for the representative coalition member is given implicitly by the condition that the marginal cost of tax and debt finance be equated, i.e.

$$f_\ell(\sum_{k \in M} g^{DRE}_k - b^{DRE}) + \mu(b^{DRE}) = 0.\quad (15)$$

(12) and (15) fully describe the unique equilibrium under the $RE$. Using (14) one immediately finds, that debt under $RE$ is greater than zero. This is intuitive, because participation uncertainty makes it a rational strategy to restrain the future government. If all decisions are taken simultaneously, the same result applies: Suppose that one minister is given the right to determine the debt-level in addition to her expenditures. With simultaneous moves, the strategies of the
other ministers cannot depend on the choice of debt but only on the expected equilibrium value of debt which is governed by (15).

In the Nash equilibrium, expenditures are inefficiently high. With debt targeting, each minister has an incentive to take the spill-overs of the other ministers’ budget proposals on her group’s welfare into account when deciding on her vote on debt. Each minister maximizes (9). The first order condition is

\[(1 - (m - 1) \gamma^D_i)\nu_g(g^{DTA}_i(b)) + \mu^{DTA}(b) = 0, \quad (16)\]

where \(\gamma^{DTA}_i \in \left[0, \frac{1}{m}\right]\) and, therefore, the term in parentheses ranges between 0 and 1. Issuing debt reduces current taxes and crowds in current expenditures. From a minister's view, only raising the output level of one's own project justifies its cost but raising the output levels of the \(m-1\) other projects implies just a cost. In itself this would give an argument for reducing debt. However, because there is a detrimental effect in the next period through \(\mu\), the selected level of debt is still positive under \(TA\), due to uncertain recognition in the second period. Comparing (14) and (16) it is straightforward that the chosen level of debt is lower under this institution than under debt renegotiation.

**Proposition 2** With fragmented decision-making, the level of debt chosen under a budget institution with debt targeting \(TA\) is at most as high as the debt-level chosen under a budgetary institution with debt renegotiation \(RE\).

Note that under fragmentation everybody in government agrees on the debt policy. Introducing a debt target setting stage gives agents an opportunity to take the other agents' reaction to a shift in
the budget constraint into account. In this sense, cutting back debt at $b_{DRE}$ offers a move to a Pareto better conjectural variations equilibrium compared to the Nash-equilibrium given by the configuration of reversion expenditures at $g_{DRE}$ where overspending prevails. The effect in the present period which calls for a reduction in debt is certain for the coalition members and, therefore, stronger than an opposite effect in the second period which calls for an extension of debt finance.

**B. Coordination of the cabinet budget**

In this section we analyze the case where one of the spending ministers also serves as budget coordinator. After the spending ministers have announced their budget proposals, the budget coordinator makes a proposal to coordinate the budget. This proposal has to be unanimously adopted. The reversion pay off for a responder is given from the policy under decentralization. The proposal right is substantial for the budget coordinator, because there are unrecognized externalities in the reversion budget policies and the budget coordinator acts as a residual claimant. As bargaining over expenditures yields a point on the Pareto frontier, bargaining over a debt target has purely distributional implications as in the parliament model.

For an ordinary member of the government, the expected utility from rejecting the budget proposal is

$$V_j^0(b) = V_j^{\sum_{i=1}^n g_i^0(b)}$$

where $g^0$ is the expenditure level under decentralized decisions. Note that $g_i^0$ and $g^0$ do not in general coincide, because expected wealth is higher when coordination takes place. As the proposal maker always makes a proposal which is immediately accepted, the probability that $g^0$ is...
actually implemented is zero. Furthermore, aggregate expenditures under coordination are lower than under decentralization.

An ordinary minister maximizes her pay off in the bargaining game when she maximizes the utility of the reservation budget. With respect to the final outcome, a strategy which raises the reservation utility has a chilling effect in the sense of Myerson (1991). In the definition of \( \mu \), we observe

\[
|\frac{\partial V_2^0}{\partial b}| < f_{\tau_2}(\tau_2). \tag{13}
\]

This is the same crowding out effect as in (13) which now works as a chilling effect in the second period. Issuing debt offers insurance against not being the proposal maker in the next period government. But this effect is mitigated to the extent that there is a chance of becoming the proposal-maker oneself.

1. **Debt renegotiation and simultaneous decisions on all variables**

With predetermined expenditures, at the financing stage preferences of all ministers coincide and the optimum level of debt is governed by the condition

\[
f_{\tau}(\Sigma_{k \in M} g_{k1} - b_{RE}) + \mu(b_{RE}) = 0. \tag{17}
\]

From (7) it is obvious that the sign of \( b \) depends on the elasticity of the reaction functions both in the actually chosen and the reversion allocation and is ambiguous. As the final allocation approaches the group efficient equal share allocation, \( b_{RE} \) is positive.\(^9\)

Next we show that there is no difference between debt renegotiation and simultaneous decisions on debt and expenditures in one legislative session. First, note that when a junior minister selects her budget demand, she maximizes her reversion utility under \( RE \) by selecting it "as though" it were financed by a debt policy which accommodates the reversion budget and which is different
from (17). Call \( b^{0\text{RE}} \) this alternative debt level which would optimally finance the reversion budget \( \{g^{k\text{RE}}\} \) if it were implemented, i.e. for which

\[
\int_a \left( \sum_{k \in M} g^{k\text{RE}}_i - b^{0\text{RE}} \right) + \mu(b^{0\text{RE}}) = 0. \tag{18}
\]

Suppose now that each junior minister not only selects a budget proposal in the first round, but also proposes a level of debt, i.e. each minister proposes \((g^0_i, b^i)\). Subsequently, the budget coordinator selects any one of the other ministers' proposals \(b^0\) and proposes \((\{g^{k*}_i\}, b^*)\) against \((\{g^0_i\}, b^i)\). Every junior minister maximizes her reservation utility \( W^0(g^i, \tau, b) \) by proposing \(b^0 = b^{0\text{RE}}\). In this case the budget coordinator can do no better than to propose \(b^{RE}\) and the RE-expenditure vector, i.e. \((\{g^{k*}_i\}, b^*) = (\{g^{RE}_i\}, b^{RE})\). This can be seen as follows: When she chooses \(g^i\) and \(b\) at the same time, \(g^i\) cannot depend on \(b\). Therefore, the first order conditions for the problem continue to be (4), (5) and (17). So the allocation is the same if the coordinating budget proposal includes expenditure and debt policies at the same time.

### 2. Setting a debt target in period one

If a debt target is selected before the reversion expenditure level is set, the junior minister compares the effects of an increase in debt in the reversion budget to the effect on the intertemporal margin. As shown in part C of the appendix, she chooses \(b^i > (<) b^{0\text{RE}}\) if

\[
(m - 1) \frac{f^0_{\tau\tau} - v^0_{gg}}{-v^0_{gg} + mf^0_{\tau\tau}} > (<) \frac{f^{0\text{RE}}_\tau - f^0_\tau}{f^0_\tau} \tag{19}
\]

for \(v(g^0_i) - mf^0_i(\{g^{k0}_i\}, b) \neq 0\) and

\[b^i = b^{0\text{RE}}\]

else. \(f^0_{\tau\tau}, v^0_{gg}\) and \(f^0_\tau\) refer to the second and first derivatives in the reversion allocation and \(f^{0\text{RE}}_\tau\) is the marginal cost of public funds in the finally accepted allocation at \(b^{RE}\). The budget
externality in the reversion budget provides an argument for reducing debt at $b^{0RE}$ and determines the l.h.s. of expression (19). As the reversion budget exceeds the ex post realized budget, the difference $(f^0_t - f^{RE}_t)$ which determines the r.h.s. is non-negative at $b^{RE}$. Financing the reversion budget calls for higher debt and this effect increases with the difference between, $f^0_t$, the marginal benefit of increasing debt with the reversion budget and the marginal benefit of increasing debt in the actually implemented budget, $f^{RE}_t$. Recall from the discussion in section V. that the utility of the junior minister is exclusively determined by her utility from the reversion budget. At $b^{RE}$ an ordinary minister has an incentive to vote for an increase in the debt target if she plans to put forward high spending demands in the subsequent budget game and the crowding-in effect on the other ministers budgets can be neglected. In the limiting case, where the reversion budget is the efficient equal distribution equilibrium, timing just has no effects on the deficit preferred, i.e. $b^j$ and $b^{RE}$ coincide. As the following lemma shows, the crowding-in effect is dominating the solution at least in the case of an inelastic marginal utility schedule.

**Lemma 1** If the elasticity of marginal utility of consuming the public good is not too great and $mf_{\tau e} \geq 1$ the debt target preferred by a junior minister under targeting is $b^j < b^{RE}$.

Proof: See appendix D.■

The lemma states a sufficient condition for the junior minister to want lower debt. In the appendix we show that under the stated condition, $b^j < b^{RE}$ holds for a small deviation from the limiting case where the coordinator proposes the efficient equal share allocation, $G^{eff}, b^{RE}(G^{eff})$. If the demand is not too elastic, particularly in the case where an increase in the utility of the responder does not decrease expenditures, it can be shown that this result must also hold for an
allocation which favors the budget coordinator. This is fulfilled if the elasticity of demand in the range \((g^j, g^\omega)\) is not smaller than \(-1\) (see part E of the appendix).

We can show (see proposition 3 below) that whenever the junior minister prefers \(b^j < b^{RE}\), the opposite is true for \(\omega\), i.e. \(b^\omega > b^{RE}\).\(^{10}\) So there is conflict over debt policies unless everybody in the government prefers \(b^{RE}\) as a debt target. Now suppose that the budget coordinator proposes a debt level against an institutionally defined reversion debt level which applies in absence of an acceptance. Acceptance has to be unanimous.\(^ {11}\) It is straightforward that if the pay off function of \(\omega\) and \(j\) were concave in \(b\), no proposal could win against the reservation value as long as the reservation value is between the individual optimum points \(b^j\) and \(b^\omega\). However, in our case, the pay off for the budget coordinator is not necessarily concave, because she claims the residual over the concave pay off for \(j\). This causes two problems: First, there might be no solution unless the utility imputation space is bounded from below (in our case by \(W(\{0\}, 0)\)).\(^ {12}\) Secondly, we cannot exclude the possibility that global optimum value \(b\) for \(\omega\) is at one of the two boundaries. Still in this case, a local result can be derived for a reservation pay off in an interval for which the problem is well-behaved, \((\hat{b}, b^j)\) or \((b^j, \hat{b})\), and where the reversion debt level is immediately accepted. Importantly, in the case \(b^{RE} > b^j\) the relationship \(\hat{b} > b^{RE}\) holds because the pay off for \(\omega\) strictly increases in the range from \(b^j\) to \(b^{RE}\) whilst \(j\)’s pay off decreases.

**Proposition 3** Suppose \(b^j < b^{RE}\). Then (a) preferences of \(\omega\) are opposed, i.e. \(b^\omega > b^{RE}\), (b) there is \(\hat{b} > b^{RE}\) such that for any reversion debt-level \(b^r \in (b^j, \hat{b})\) the realized debt target is \(b^{TA} = b^r\), (c) if \(b^\omega\) is a global optimum, the maximum value of the reversion level which gets accepted is \(\hat{b} = b^\omega\).

Proof: See part F. of the appendix.■
With this proposition we have a local property of stabilization programs which take the form of a switch to debt targeting procedures. If the agenda determines a reversion debt level $b^j \leq b^\omega < b^{RE+\varepsilon}$, $\varepsilon > 0$, then this is always accepted as the debt target. If $b^\omega$ is also a global maximum, this results holds for any reversion level between $b^j$ and $b^\omega$. Only $b^\omega$ is not a global maximum, there is an upper bound $b^u$ on $b^\omega$ where $\omega$ proposes $b^{TA} < b^\omega$.

There is an interesting policy implication here for the case where a debt level is not specified in the agenda but instead is given endogenously by the debt level which would be realized under renegotiation. In that case, the acceptable debt target is not more ambitious than what would be realized under debt renegotiation anyway. If the government budget is efficient, the only function of a debt target as opposed to renegotiation can possibly be to shift bargaining power in the budget negotiations. Therefore, there is necessarily conflict over the target.

VII. Vote buying in the Congress

Expenditure decisions in the US congress are dominated by committees. Expenditure laws are adopted by coalitions of varying composition. Furthermore, arrangements in the congress allow to sustain package deals which are said to result in universalistic outcomes (see Shepsle and Weingast, 1981 and Weingast and Marshall, 1988) A model which accounts for these features is the vote buying scenario of Chari and Cole (1995).

In the vote buying model, proposals are decentralized. When submitting a proposal, each legislator $i$ maximizes the surplus of her utility from her preferred project over the compensation
payments to other legislators. Another legislator $j$ votes in favor of the proposal, if she is compensated for the tax payments incurred by the project. Introducing compensation payments from $i$ to $j$, $t_{ij}$, the maximization problem of $i$ for a given debt level and given expenditure demands $g_k^{kU}$ of the other legislators is

$$\max_{g_i', t_{ij}', j \in C, \{g_k^{kU}\}_{k \neq N\backslash i}, \bar{b}} W_i'(g_i', \{t_{ij}'\}_{j \in C}, \{g_k^{kU}\}_{k \neq N\backslash i}, \bar{b}) \text{ s.t. } W_i' \geq W_i'(0,0,\{g_k^{kU}\}_{k \neq N\backslash i}, \bar{b}) \quad i = 1, \ldots, n.$$ 

The pivotal equilibrium established by Chari and Cole is characterized as follows: each legislator gets her proposal passed; for each proposal $i$, there are $m$ legislators who receive $t_{ij} = g_i$ and the level of each project level is given by

$$v_i(g_{U}^{i}) = mf_i \left( \sum_{k \in N} g_k^{kU} + (-1)^i \bar{b} \right)$$

(20)

Each project is run at a socially inefficient level because only the cost $m$ legislators are internalized. Any single expenditure level is lower than in the cabinet with fragmented decisions, where no internalization takes place. The expected monetary net payment for each legislator is zero, as is easy to see: each legislator spends money on $m-1$ other legislators. She potentially receives money from $n-1$ other legislators, each of which chooses $i$ with probability $(m-1)/(n-1)$. The effect of debt policies is similar to the effect in the decentralized decision framework of the cabinet without coordination. The cost of debt finance, which is found in analogy to (14) is (see part G of the appendix).

$$\mu_v = \left( \frac{1}{m} - \left(1 - \frac{n}{m}\right) \gamma_2^U \right) v(g_z^U).$$

(21)

with crowding out $\gamma_2^U \in \left[-\frac{1}{n}, 0\right]$. From (20), the cost of one unit of debt is $\frac{1}{m} v_i(g_{U}^{i})$. Crowding out reduces this cost because the benefit of each unit in the project foregone is only $v_i$ whereas...
the social costs are \( n/m \) times the benefit. The solution to the debt targeting problem, in analogy to (16) now reads

\[
\frac{1}{m} - (1 - \frac{n}{m}) \gamma^U_i \nu(g_i^{UTA}(b)) + \mu^{UTA}(b) = 0.
\]

with \( \gamma^U_i \in \left[ 0, \frac{1}{n} \right] \). Debt has a crowding-in effect in the first period. As the social cost of expenditures exceed the expected benefit, this calls for a reduction in debt. The efficiency effects in the first and second period balance each other at \( b=0 \). It is perhaps not surprising that universalism calls for a materially balanced budget. What drives this result, however, is the congruence of expectations for the first and second period. If not all projects were realized, but each legislator would face the same risk of failing with her project, the debt policy selected would be just the same.

On the other hand, debt renegotiation would yield positive debt, because the ex ante efficiency consideration suggests a policy which crowds out inefficiently high spending in the next period. Under \( RE \), the condition governing debt policy is

\[
f_i \left( \sum_{k \in N} g^U_{1k}^{RE} - b \right) + \mu^{RE}(b) = 0.
\]

which is satisfied for \( b^{RE} > 0 \).
VIII. Conclusion

I have compared debt and expenditure policies in different institutional settings. The model explains observed policy differentials between different countries: In the congressional system it is most likely that an agreement on a zero debt target will be reached if such a move is on the floor. It also explains a coalition party's incentive over time to defect on a formerly agreed debt target. A debt target might work because of the following reasons: (1) It has a strategic effect on the reversion pay off for the junior coalition members where the decisions are made in cabinet. Therefore, it gives rise to a distributional conflict between the budget coordinator and other coalition members. (2) Issuing debt interferes with the opportunity set of the coalition which is to emerge at the next stage of the bargaining game. If the bargaining powers are equally distributed in the future period as they are in the targeting stage, this gives an ex ante justification for accepting a materially balanced debt target.

In both cases, and rather trivially, if a debt target is effective it is also vulnerable to renegotiation. If the expenditure budget is efficient, the only function of a debt target is to shift bargaining power within the coalition. A consequence is that if the institution does not specify a reversion debt level which will prevail in the absence of an agreement, coalition members will only be able to agree on a debt target which is ineffective. While externally imposed deficit targets have been seen as unjustified (see, for example Buiter, 2005) the inability to internally agree on an effective debt target is worrying in our setting because any positive debt emerges entirely for strategic reasons and the debt policy under renegotiation serves the interest of the present coalition government at the expense of future governments.
References


Appendix

A. Concavity of V

In this part of the appendix I show that $\mu'(b|\psi^D)<0$ and $R_{bb}<0$.

$$\mu'(b) = V_{bb} = P(g_b)^2 v_{gg} - (\tau_b)^2 f_{\tau\tau} + Z$$

where $P = \frac{m}{n}$, $Z = v_g(Pg_{bb} - \tau_{bb})$ and $g_b = -\frac{f_{\tau\tau}}{\Delta}<0$, $\tau_b = 1 + mg_b \in [0,1]$ and

$$\Delta = - v_{gg} + mf_{\tau\tau} > 0.$$ In the expression for $V_{bb}$, the first two terms are negative and only the third term in the sum, $Z$, is ambiguous. Next, differentiating $f_{\tau\tau} \tau_b - v_{gg} g_b$ and $\tau_b - mg_b = 1$ one gets

$$g_{bb} = \frac{1}{\Delta} \left[ (g_b)^2 v_{gg} \right]$$

$$\tau_{bb} = mg_{bb}.$$ where we have made use of the assumption that $f_{\tau\tau} = 0$. Now one can rewrite

$$Z = \frac{v_g(P-m)}{\Delta} \left[ v_{gss} (g_b)^2 \right]$$

The first multiplier is negative, for $P<1$ and/or $m>1$ and the bracketed term non negative for $v_{gss} \geq 0$, so this proofs $\mu'(b|\psi^D)<0$. Concavity of $R$ can be shown along the same line.

B. Derivation of equation (14)

Differentiating (6) gives for a coalition member $j$: $V^j = \gamma v_g - m f_{\tau}$ and for non member $k$: $V^k = - (\mu \gamma + 1) f_{\tau}$. Using $f_{\tau} = v_g$ and taking expectations yields

$$\mu = \frac{m}{n}(\gamma - m\gamma - 1)v_g - \frac{m-1}{n}(m\gamma + 1)v_g.$$ Rearranging yields (14)

C. Derivation of Inequality (19)

The effect of an increase in $b$ on the reversion utility is given by

$$R^0_b = \left(v_g (g_1^0) - mf_{\tau} (\{g_1^0\}, b)\right)\psi^0 + f_{\tau} (\{g_1^0\}, b)$$

(22)
where \( \varphi^0 : \frac{dg^0}{db} = -\frac{f^0}{\gamma_{gs} + mf^0} \) is the crowding-in effect of public expenditures in the reversion budget at the current stage. The ordinary minister prefers an increase in debt, if at \( b^{RE} \) the condition \( R^0_\gamma + \mu(b) > 0 \) holds and prefers \( b^{RE} \) if the same condition is fulfilled as an equality (sufficiency follows from appendix A). We know that at \( b^{RE} \), \( \mu(b^{RE}) = -f^\tau(x, b^{RE}) \) because, if the subsequent proposal by \( \omega \) is going to be accepted, then second period effects of debt finance are given by \( \mu(b^{RE}) \). Furthermore, \( v^0_g = f^\tau \) in the reversion allocation. It is straightforward to show that the condition in lemma 1 is equivalent to the condition that the ordinary minister wants a higher debt.

In the efficient equal share equilibrium, \( v^0_g(g^{i,0}(b)) - mf^\tau(x, (g^{i,0}(b), b) = 0 \), so \( R^0_b = f^\tau(x, (g^{i,0}(b), b) \) and \( (g^{i,0}(b)) = g^{\tau,0}(b) \). The latter claim is true, because, if the reversion budget is the efficient equal share budget, the proposal maker can do no better than to propose the same budget at the expenditure negotiation stage. Thus, the debt level \( b^{RE,eff} \) which finances the efficient equal share allocation is unanimously preferred under \( TA \).

**D. Proof of lemma 1**

To proof the lemma we show that condition (19) for \( b^i < b^{RE} \) holds if \( G^{RE} \) is arbitrarily close to \( G^{eff} \) which implies that \( G^0 \) approaches \( G^{eff} \) from above. If \( G \) decreases with a rise in equality, i.e. \( G^{RE}(b^{RE}) < G^{eff}(b^{RE}) \) in the allocation governed by (4) and (5), \( f^{\tau,RE} < f^{\tau,eff} \) and all results extend to the inequitable allocation.
Define \( X = \frac{f_{\tau}^0}{v_{gg}^0 + mf_{\tau}^0} \) and \( Y = \frac{f_{\tau}^0 - f_{\tau}^{\text{eff}}}{(m - 1)f_{\tau}^0} \). At \( v_{gg}^0 = 0 \) we have \( \frac{(m - 1)}{m} > \frac{f_{\tau}^0 - f_{\tau}^{\text{eff}}}{f_{\tau}^0} \) if \( \frac{f_{\tau}^{\text{eff}}}{f_{\tau}^0} > \frac{1}{m} \).

At \( G^0 \), the condition \( f_{\tau}^0(G^0, b^{RE}) = v_{g}^0(g^0) \) holds and at \( G^{\text{eff}} \) the condition

\[
mf_{\tau}^{\text{eff}}(G^{\text{eff}}, b^{RE}) = v_{g}^0(g)\]

holds. \( f_{\tau} \) increases in \( G \). Because the cost curve \( mf_{\tau} \) is above \( f_{\tau} \), \( g^0 > g^{\text{eff}} \) and, consequently, \( mf_{\tau}^{\text{eff}} > f_{\tau}^0 \). Therefore, at \( v_{gg}^0 = 0, X > Y \). At \( \lim_{v_{gg}^0 \to \infty} \), the l.h.s. of expression (19) is 0. The r.h.s. is also zero because \( G^0 = G^{\text{eff}} = G \) and, therefore, \( f_{\tau}^0 = f_{\tau}^{\text{eff}} = f_{\tau}^{RE} \).

Finally, we have to show that \( Y \) does not cut \( X \) from below. This is the case if \( \frac{\partial Y}{\partial |v_{gg}^0|} \geq \frac{\partial X}{\partial |v_{gg}^0|} \), i.e. the r.h.s. is steeper. We have \( \frac{\partial X}{\partial |v_{gg}^0|} = -\frac{1}{|v_{gg}^0|} \). To get an expression for the other derivative

and for \( v_{gg}^0 > 0 \) we have to determine the difference in taxes between both allocations, \( \Delta \tau \).

\[
v_{g}(g) = f_{\tau}(G^0, b^{RE}) + \Delta g|v_{gg}| = mf_{\tau}^0(G^0, b^{RE}) + mf_{\tau} \Delta g \]

where \( \Delta g \) is the absolute value of the difference between \( g^{\text{eff}} \) and \( g^0 \) and we have linearized the demand schedule at \( g^0 \). This gives an upper limit, \( \Delta g \leq \frac{(m - 1)f_{\tau}(G^0, b^{RE})}{|v_{gg}| - mf_{\tau} \Delta \tau} \). Finally, \( \Delta \tau \geq m \Delta g \). Note that with a convex demand schedule, using \( \Delta g \) to determine the final allocation tends to underestimate \( G^{\text{eff}} \). Now,

\[
\frac{\partial Y}{\partial |v_{gg}^0|} \leq \frac{\partial \Delta \tau}{\partial |v_{gg}^0|} \frac{1}{(m - 1)f_{\tau}(G^0, b^{RE})} = \frac{1}{mf_{\tau}^0} \frac{1}{|v_{gg}|}. \]

From this, with \( f_{\tau} \geq 1 \) and \( m > 1 \) follows

\[
\frac{\partial X}{\partial |v_{gg}^0|} \geq \frac{\partial Y}{\partial |v_{gg}^0|} \]

and \( Y \) does not cut \( X \) from below.

**E. Marginal expenditure effects of an increase in equality**

Using (4) and (5), consider the expression for aggregate expenditures as a function of \( \lambda \):

\[
G(\lambda) = g^\omega + (m - 1)g^\omega = v_{g}^{-1}(1 + \lambda) + (m - 1 + \frac{m - 1}{\lambda})).
\]
Using \( v^{-1}(a \lambda) = \frac{a}{v_s} \) one gets

\[
\frac{\partial G}{\partial \lambda} = \frac{1}{v_s(g^\omega)} - \frac{(m-1)^2}{m^2} \frac{1}{v_s(g^\prime)}
\]

(23)

Noting that the f.o.c.’s imply that \( \frac{\lambda}{m-1} = \frac{v_s(g^\omega)}{v_s(g^\prime)} \) and using the elasticity of marginal utility of consumption \( \eta(g) = -\frac{v_g(g)}{v_s(g)} \) one finds that \( \frac{\partial G}{\partial \lambda} \geq 0 \) if

\[
\frac{g^\prime / \eta(g^\prime)}{g^\omega / \eta(g^\omega) \geq \frac{\lambda}{m-1}}.
\]

(24)

For \( g^\prime = \alpha \frac{\eta(g^\prime)}{\eta(g^\omega)} g^\omega \) where \( \alpha = \frac{\lambda}{m-1} \) the f.o.c. (4) and (5) are fulfilled but condition (24) would only hold as an equality. Consequently, \((g^\omega, g^\prime)\) simultaneously satify the f.o.c.’s and (24) if at \((g^\omega, \alpha \frac{\eta(g^\prime)}{\eta(g^\omega)} g^\omega)\) we have \( v_s(\alpha \frac{\eta(g^\prime)}{\eta(g^\omega)} g^\omega) \leq \frac{1}{\alpha} v_s(g^\omega) \). This claim is true if the elasticity of demand in the range \((g^\prime, g^\omega)\) is not smaller than \(-1\).

**F. Proof of proposition 4**

In order to prove (a), from the problem of \( \omega \) one gets

\[
V_b^{(b)}(b) = \int f_i(\sum_k g_i^k(b) - b) + \mu(b) - \lambda(R^0_b + \mu(b))
\]

At \( b^{RE} \), \( f_i(\sum_k g_i^k(b^{RE}) - b^{RE}) = -\mu(b^{RE}) \). First, consider \( b^i < b^{RE} \). For \( b < b^{RE} \), it must be that

\[
f_i(\sum_k g_i^k(b) - b) > -\mu(b) \text{ because } \frac{\partial g}{\partial b} < 1. \text{ Because at } b^i, R^0_b(b^i) + \mu(b^i) = 0, \text{ the budget}
\]

coordinator wishes the debt to increase. The bracketed term following \( \lambda \) is positive for \( b > b^i \), i.e.
for $R_b^0 < -\mu$ with $-\mu_b > 0$ and $R_{\mu_b}^0 < 0$, so $V'_{b^\omega}(b) > 0$ for $b \in [b', b^{RE}]$. The same argument can be repeated for the case where $b' > b^{RE}$.

$b^\omega$ must at least be a local maximum. In figure 2 I have sketched the objective functions for $j$, $V^{\theta_j}(b)$ and for $\omega$, $V^{\omega}(b, S(b))$ which is the residual for $\omega$ after satisfying $V^{\theta_j}(b)$. $V^{\omega}(V^{\theta_j}(b^{RE}))$ gives the value of the objective function for $\omega$ if $V^{\theta_j}$ is fixed at its value for $b^{RE}$. The function $V^{\omega}(b)$ is concave in $b$ and maximized under a concave constraint $V \geq S(b)$ where

$S(b) = \text{Max}(V^{\theta_j}(b), W(\{0\}, 0))$ (we had assumed that $W(0,0) = 0$). To prove (b), note that $b^{RE}$ is a maximizer of the problem $\max_b V^{\omega}(b)$ s.t. $V \geq V^{\theta_j}(b)$: from concavity of $V^{\theta_j}(b)$ one can construct $b'' < b'$ such that $V^{\theta_j}(b'') = V^{\theta_j}(b^{RE})$. Obviously, $V^{\omega}(b^{RE}, S(b^{RE})) > V(b, S(b))$ for $b \in (b'', b^{RE})$, i.e. for $\omega$, $b^{RE}$ dominates all values in the interval. As the constraint is strictly concave in $(b', b^{RE} + \epsilon)$, the domination is strict and it must be true that $\Delta = V^{\omega}(b^{RE}) - V^{\omega}(b'') > 0$. Then from continuity of the constraint, there must be a pair $(b', b'''|b' > b^{RE}, b''' < b', V^{\theta_j}(b') = V^{\theta_j}(b'''))$ such that $V^{\omega}(b') > V^{\omega}(b''')$. Define $\hat{b} = \min \{b^{\omega}, b^{\max} = \max_b (V^{\omega}(b'), V^{\omega}(b''''))\}$, i.e. $\hat{b}$ is the largest value for which a pair $(b', b''')$ can be constructed or the smaller value $b^{\omega}$ which is the local maximizer of $\omega$'s problem. Then, by construction, as long as $b' \in [\hat{b}', \hat{b})$, $b'$ prevails: if $\omega$ has to propose $b$ to be voted against $b'$ such that the reversion utility level is $V^{\theta_j}(b')$ then there is no $b \neq b'$, such that $V^{\omega}(b) > V^{\omega}(b')$ and $V^{\theta_j}(b) \geq V^{\theta_j}(b')$. Finally, if $b^{\omega}$ is a global maximum, $\hat{b} = b^{\omega}$ from its definition.

G. Derivation of equation 21

Differentiating (6) gives for each agent: $V_b = \gamma v_g - f \gamma n - I$. Using $f = \frac{V_g}{m}$ this yields which,

$V_b = \gamma v_g - (v_g/m) \gamma n - (v_g/m)$ which after rearranging, gives 21.
RE-timing
reservation allocation (exogenously given or decentrally chosen)
\{g_{\omega}^i\}_{i=1}^{n} \quad \{g_{\omega}^a, g_{\omega}^j\}, j \in C \not\in \omega
parliament \{0\}
cabinet \{g_{\omega}^0\}
universalism \{g_{\omega}^U\}

ω proposes
expenditure vector

ω proposes
finance mix

b, τ
all institutions

TA-timing
ω proposes debt target (exogenously given or decentrally chosen)

b against b′

all institutions

\{g_{\omega}^i(b)\}_{i=1}^{n} \quad \{g_{\omega}^a(b), g_{\omega}^j(b)\}, j \in C \not\in \omega
parliament \{0\}, f(−b)
cabinet \{g_{\omega}^0(b)\}
universalism \{g_{\omega}^U(b)\}

Figure 1: Overview of the timing of events

Figure 2: Bargaining over a debt target
Endnotes

1 As I consider a model where all agents have equal power ex ante it is straightforward to show that there is no difference between RE and completely simultaneous decisions on tax, expenditures and debt.

2 For a representation with forward looking consumer behavior see for example Persson (1998).

3 For convenience I suppress the index t where no confusion can arise.

4 \( W(\emptyset, .) \) may be justified as the outcome which is implemented by a caretaker government which observes the obligation to pay off the debt in the second period.

5 A formal framework in which such a distribution of portfolios is the outcome of a bargaining game is presented in Pech (2004). This framework assumes arbitrary recognition of the head of government. It allows for equal shares in the government (i.e. one ministry per party) and at the same time for different output levels (i.e. the head of government may realise a higher output in his ministry). In Baron (1991), shares in the government differ but output levels coincide.

6 There is one annotation due in view of the results which we find for the expenditure negotiation stage. In principle, a formateur at the government formation stage could feel tempted to offer jurisdictions to more than \( m-1 \) parties if including more parties has a negative effect on their common threat point in the budget game. In the following, we rule out this possibility.

7 Informal budget rules are often more important in practice than formal rules. For example, when the Belgium budgeting system shifted from a (decentralised) procedure where the minister of finance simply collected the proposals of the other cabinet ministers towards a (centralised) procedure where the minister of finance makes a proposal for each jurisdiction. Yet in practice the other ministers where still free to accept their share or to argue that they needed more funds. In the end, outcomes did hardly change. I am grateful to George Stienlet for his insights of the Belgium budgeting system.

8 Nash behavior in the first stage is still optimal even for \( \omega \) if we assume discretion in the execution of the budget. Under discretion, an announcement of \( \omega \) in the initial budget proposal to overspend which she might use to curtail the other ministers' demands is not credible.

9 In that case, the problem is one of tying the hands of a monolithic successor government as in Persson/Svennson (1989).
10 For the case $b^i > b^{RE}$ we similarly show that $b^{\omega} < b^{RE}$ and all our results extend accordingly.

11 If a vote were taken in the parliament with a simple majority, the opposition would vote with the party of the minister who wants a lower debt level.

12 In the problem for the budget coordinator $\omega$, the degree of convexity of the constraint $W^\omega(b)$ exceeds the degree of concavity of the unconstrained objective function $W_\omega(b)$. Therefore, returns for $\omega$ could become unbounded even though the unconstrained objective function is bounded from above.

13 Chari and Cole also consider a game with side payments by the constituents instead of side payment among legislators to show that negligible payments are compatible with an equilibrium.