Why Does the Government Obey the Constitution? -
Theory and Application to Tax Evasion*

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ABSTRACT

Why does the government not defect from the constitution? This article focuses on the dynamic restraints for the government, which result from the rule of law: violations against unconstitutional laws are not punished under the constitution. If the government is opportunistic and cannot commit itself to stay outside of the constitution once it has defected any attempt to enforce an unconstitutional law is hampered. Indeed, citizens’ expectations to go unpunished when not complying can be self-fulfilling. This considered, a government is deterred from defecting from a constitutionally specified tax rate which it otherwise would.

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Why is the constitution sustainable? Obviously, it is just paper work and the government, in the absence of an exogenous enforcement mechanism, could easily do away with it! Is there any endogenous enforcement mechanism, which would deter the government from doing so? One formalised approach which suggests that there is such an intrinsic force was given by Kotlikoff/Persson/Svensson [1988]. Basically, their argument is that as long as anybody else obeys the social contract, everyone is better off. In their approach, any temptation to violate the social contract in the short run is counterbalanced by the long run adversary effects which in turn depend on the expectations of the citizenry. However intuitive their basic message, their setup depends crucially on an exogenous punishment period and with that on rather strong informational requirements. Also, the superiority of the contract bases on the superiority of obeying a rule as compared to discretion. There is nothing in their argument which points to a feature which is unique to constitutional law.

Our own approach is a completely new way to deal with the problem. It combines two ideas which we believe to be crucial in the sustainability of the constitution. First, the citizens can, by non complying behaviour, incur a heavy cost on the government. Examples include the refusal of young Spanish men to be drawn to the conscription army and the British poll tax revolt. In the former case, the fact that up from a certain point the sheer number of violations rendered the individual prospects of getting punished rather small certainly helped the success of the uprising.

The second idea refers to a dynamic restraint which we find to be present in any constitution, which we know: by the principle that the constitution establishes a state under the
rule of law, there is no punishment of violations of non constitutional law under the constitution. Such a promise should give an incentive for citizens to engage in non complying behaviour. Whereas a non constitutional government cannot credibly commit to behave in a non opportunistic way, the constitutional one can and, thereby, encourage citizen’s opposition to non constitutional governments. This points to the wider context of illegitimacy of the behaviour of the executive power, which has explicitly been exploited within the constitutional framework to rein lower levels of the government. For example, after the Vietnam war an amendment has been introduced in the code of conduct for US service personnel which requires that soldiers only execute commands which are in accordance with international law.\(^1\)

Putting things together, we can show that there are cases where the constitution is sustainable precisely because it promises impunity and, thereby, encourages non complying behaviour towards non constitutional governments. While we believe that our point is of wider applicability, we formalise our concept in the framework of a fiscal constitution. Apart from establishing a state under the rule of law, i.e. promising impunity for trespasses against non constitutional laws, a constitution simply consists of a maximum constitutional tax rate, which is exogenously given. The government, which is modelled as a revenue maximiser,\(^2\) can try to tax people more heavily but, in order to do so, has to violate the constitution. Citizens, on their part, decide on whether to declare income for tax purposes or to evade taxes. The

\(^1\)We are grateful to Michael Chwe for making us aware of this point.

\(^2\)Our positive model of government suits the logic of Leviathan in Brennan/Buchanan [1980], but only with respect to the logic of governmental decision making. Our constitutional approach is totally different to that of Brennan/Buchanan [1980].
government enforces its tax law by undertaking costly detection efforts and charging a fine on detected evaders.

There is always the possibility that the government decides to switch back to the constitutional state and adopt a constitutionally admissible tax law. By doing so, it sacrifices income from fining evaders and that share of tax receipts, which are considered illegal under the constitution. Clearly, if the citizens believe, that the government carries out the announced punishment, both under the constitution and under the non constitutional regime, the non constitutional regime is always better for the government, because it acts without constraint. On the other hand, once the citizens feel that the punishment in the non constitutional state is uncertain, because the government might decide to switch back, the non constitutional government faces a problem. Citizens will discount the probability of getting fined for attempting tax evasion and, as a consequence, evasive behaviour will increase. We show, that if the enforcement cost of the government is sufficiently increasing in the share of tax evaders, there is an equilibrium, where everybody believes that the government switches back and the government, confronted with high detection costs and low tax revenue, decides that this is the best thing to do. In this equilibrium, expectations of the citizens are self fulfilling.

While the existence of a multiplicity of equilibria is interesting, it is not a fully satisfactory result. To solve the indeterminacy problem, we introduce a variable $k$, which represents the stubbornness of the government to stay in the non constitutional state, or its preference to violate the constitution. This, in the end, governs the government’s decision whether to switch back or not. We assume that this variable can be observed by the citizenry with noise
and that the noise generating process is common knowledge. In this case, the situation can be analysed as a global game and has a unique equilibrium point \( k^* \). A government, whose stubbornness is below \( k^* \), is forced to switch back to the constitution.

With \( k^* \) determined for the switch back decision in the non constitutional state, we can ask, under what circumstances a constitutional government is prepared to give up or stay with the constitution. Basically, the government has to be aware that once it has defected from the constitution, its "future self" could feel tempted to undertake a switch back, if its stubbornness is not strong enough. Whenever the government knows for sure, that it has to switch back and sacrifice some of its revenue, it does better by staying with the constitution at the outset. But even in those cases where the government only puts a small probability on the event of a switch back, the incentive to violate the constitution in the first place is definitely weakened.

For analysing this case, we assume that the preference for violating the constitution, \( k \), is given at any point in time but is subject to an exogenous stochastic process. Both, the government and the citizens, know the precise value of \( k \) at any point, except that citizens only observe a noisy signal when they are making their decision on tax declaration or evasion. By this assumption we rule out more complex issues arising in situations, where the government has an opportunity to influence the citizen’s beliefs by the actions it takes. Our findings are: Given a preference in the constitutional state, \( k_{t-1} \), there exists a corresponding switching point, which applies after the constitution has been violated. There is a critical preference in the constitutional state, up to which the government stays with the constitution. When the switch back mechanism applies, the critical preference for violating the constitutional state

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is higher than in a situation, where the government is able to commit not to switch back.

Our top down approach differs from the mainstream of both normative and positive constitutional analysis, in that there is no hypothetical or factual constitutional decision making by citizens. Historical experience suggests that the point we make has empirical relevance, because dictatorial usurpers indeed undertake great efforts and spend resources to making their announcements of punishment credible. Furthermore, we often find unconstitutional usurpers to be keen to mask the fact that they are violating the constitution, for example by keeping to procedural requirements established in the constitution.

Our argument contrasts to the usual collective action arguments brought forth in order to explain constitutional enforcement like revolution or self-protecting agencies. The concept of beliefs underlying our model is also quite different to models of herding behaviour [Banerjee, 1992]. The latter rely on observability of the fellow citizen’s actions which is an assumption less defensible in a tax evasion framework. The same is true of the signalling model employed by Susanne Lohmann [1993] to explain the East German revolution. When solving the game under imperfect information, we draw on results of Morris and Shin [1998, 1999, 2000] who show the existence of a unique equilibrium in a global game in a macroecomonic context. Pech [2001] has extended their results to global games with heterogenous citizens. The implied distribution of individual cut off points bears some similarities to threshold models of collective action [Yin, 1998]. However, our ”thresholds” arise endogenously from the model. Furthermore, we also present arguments, that more dispersed societies are more prone to unconstitutional usurpation of power where we measure dispersion as differences in the citizens’ tax honesty.
Our paper is organized as follows: In section 2 we discuss the whole structure of the political game. In section 3 we set out a simple model of tax evasion. The problem of tax evasion in the constitutional and non constitutional state is analyzed in more detail in section 4. We show, that under certainty there exist multiple equilibria. In section 5 we derive a unique equilibrium for the game, which unfolds after the violation of the constitution, where information of the government’s preferences is imperfect. Section 6 considers the impact of the dispersion of the citizen’s preferences and derives the critical value for the government’s preference to violate the constitution at the outset. Section 7 concludes the paper.

II. The Game Structure

Figure 1 depicts the basic game between subsequent governments and the citizens. Node c signifies the constitutional and node nc the non constitutional state. Suppose that in stage 0 in period t, the government is in the constitutional node \((c_t^0)\). At stage 0 in t, it chooses a tax rate \(\tau\) and a level of detection effort \(r\). It does not carry out its detection effort, however, before citizens have declared their income.\(^4\) If the tax rate announced exceeds the constitutionally admissible rate \(\tau^c\) it moves to the non constitutional node. Citizens observe the tax rate and declare their income at stage 1. This is equivalent to the citizens on aggregate announcing the share of tax evaders, \(\theta\). If it is on the constitutional branch, however, there is no further decision to be taken and the government simply carries out its chosen policy. If it is on the non constitutional branch in stage 2, it observes the share

\(^4\)In our model, all tax payers have the same income. Declaring income is substantial, however, if there are rich and poor citizens and taxing the poor is impossible. A rich person would then have to declare herself to be poor in order to go without taxes.
of evaders and either enforces its tax policy with the pre-selected effort\textsuperscript{5} or chooses not to enforce its policy but instead to switch back to the constitutional state in which case it does not collect any fines and collects only that share of taxes which is in accordance with the constitution, i.e. which results under a tax rate of \( \tau^c \).\textsuperscript{6} After having chosen to switch back, the government can expect a certain pay off from taxing at the constitutional rate in \( t + 1 \).

If the government is in the non constitutional state, in \( t + 1 \), the non constitutional tax rate still prevails and citizens declare taxes accordingly.

Insert figure1 about here

For their tax evasion decision in \( nc^1_t \), citizens have to assess the probability, \( P \), of a switch back taking place. We derive two equilibria under certainty: One, where \( P = 1 \) and the government switches back. And one, where \( P = 0 \) and the government stays in the non

\textsuperscript{5}The assumption that the government precommits to a certain level of detection effort but is free to scrap detection altogether can be justified by arguing that the government has to acquire personnel in advance but has the option to sack its employees once it switches back. This assumption tremendously simplifies the exposition of the model without changing the basic results. Note that the calculations in the examples are carried out under the assumption that the government knows exactly the expectation of the citizens in which case precommitment to effort is without consequence. Therefore, the examples show that even if the government realises the a "fine-tuned" effort level it may still wish to switch back.

\textsuperscript{6}A move back to the constitutional state would at least give the citizen a right to claim back what the government has received before, so we have to take this issue into account. Note that the right to claim back excessive taxes works against the sustainability mechanism on which this model relies. If we were to extend the model to several periods, unlimited rights to claim back taxes would definitely bring the mechanism to a stop when the government has lived through several periods of non constitutional behaviour. However, in fiscal practice it is quite common to have deadlines, which make the the tax bill unrefutable, even in cases where a tax law is annulled by the constitutional court.
constitutional state.

We extend our model to cover the case of uncertainty about one fundamental value \( k \), which represents the stubbornness of the government in violating the constitution even if it looses money by doing so. We assume that the government learns the value of \( k \) before it decides on whether or not to switch back whereas citizens have to decide on tax evasion when they only observe a noisy signal of \( k \). We show, that there exists a unique value \( k^* \), such that the government switches back if \( k < k^* \).

With a model yielding a unique equilibrium for the non constitutional branch it makes sense to think about the government’s decision in stage \( c_0^t \) on whether or not to violate the constitution. We show, that if \( k_t \) results from a commonly observable variable \( k_{t-1} \), then there exists a unique value \( k_{t-1}^{**} \), such that the government does not violate the constitution, if \( k_{t-1} < k_{t-1}^{**} \).

Clearly, our result that the constitutional state might be preferred by the government above the non constitutional state hinges on the assumption that the government is effectively able to commit itself not to defect from the constitutional state once it has chosen to act within the constitution for a given period. More specifically, it does not defect after citizens have declared their income putting trust in the constitution. The alternative assumption would be that the government can defect from the constitution after citizens have announced their taxable income. Suppose that defection were indeed an equilibrium. As the government would always defect ex post, citizens would declare income as if the non constitutional state would apply. As the government cannot commit itself to obeying the constitution once it has switched back, and violating the constitution is an equilibrium, the government would
effectively find itself in the non constitutional state at any point in time. The assumptions on the timing of our model which we subsequently develop involve that the citizen, after having declared and paid taxes leaves his money in his bank account long enough such that he could pay an eventual fine.\footnote{This results from our assumption that the fine is restricted to current income.} For this spell of time, he would effectively be vulnerable to the government’s reneging on the tax rate. Such renegotiation, however, would be essentially a capital levy and should be dealt with in a model where capital formation is explicitly taken into account.

III. The Basic Model of Tax Evasion

In this section we set out a basic model of tax evasion which is a variant of Kolm [1973]. We normalise gross income and population size to one, denominate $y^i$ the income declared for tax purposes by citizen $i$, $r$ the probability of detection and $\tau$ the tax rate. A fine $\Phi$ is levied at a linear rate $\Theta$ on non declared income $1 - y^i$ and satisfies the (period by period) non-bankruptcy condition, i.e. $\Theta(1 - y^i) + \tau y^i \leq 1$ for all $y^i$.\footnote{As all citizens - including the marginal citizen who does so by assumption - realise a corner solution, the restraint is, effectively, $\Theta(1 - y^i) \leq 1$.}

A. The Citizen’s Problem

Citizen $i$’s utility is linear in disposable income, i.e.

$$U^i = 1 - (\rho^i \tau + (1 - \rho^i)\tau^c - \eta^i) y^i - \rho^i \tau \Theta(1 - y^i).$$

(1) $\rho^i = (1 - P^i)$ is the citizen’s subjective probability that the government enforces its policy.
In the constitutional state, the policy is always enforced, i.e. \( \rho^i = 1 \) for all \( i \) but in the non constitutional state, enforcement depends on whether or not the government will stick to the non constitutional state. With \( \rho^i = 1 \), \( \tau^iy^i \) is the expected monetary sacrifice when an income of \( y^i \) has been declared for taxes. In the non constitutional state, the announced tax rate \( \tau \) and the constitutional tax rate, \( \tau^c \), differ. \( \rho^i\tau + (1 - \rho^i)\tau^c \) is the expected tax rate when the citizen is uncertain on whether the announced or the constitutional tax rate applies. \( \eta^i \) is the citizen’s degree of tax honesty which takes the form of a pleasure (\( \eta^i > 0 \)) or disutility (\( \eta^i < 0 \)) derived from declaring income. \( \rho^i r(1 - y^i) \) is the expected monetary disutility from not declaring income \( (1 - y^i) \). Utility is linear in \( y^i \) so the citizen’s tax evasion decision takes him to a corner solution. He declares taxes if \( \eta^i \geq \rho^i r(1 - \rho^i)\tau^c - \rho^i r\Theta \), where equality results from an assumption which we set to break indifference.

**Lemma 1** Unless citizens are indifferent to the tax evasion problem (in which case we assume that they declare their whole income) they either declare their whole income or nothing at all. Citizens declare taxes if \( \eta^i \geq \rho^i\tau + (1 - \rho^i)\tau^c - \rho^i r\Theta \).

**Proof.** See discussion above. ■

One implication of our assumptions is, that tax evasion increases in the expected tax rate. For the conditions, under which such a reaction function can be obtained from a model with risk averse citizens see Allingham and Sandmo [1972] and Yitzhaki [1974].

**B. The Government’s Problem**

We analyse the government’s problem to select \( \Theta \), \( r \in [0, 1] \) and \( \tau \in [0, 1] \) such that revenue is maximised when it is in the non constitutional branch. It has to observe the effect on
the share of tax evaders, $\theta$, and on detection costs, $C(r, \theta)$, which we assume to be convex and increasing in both arguments. For a given government income from fines it is always better to increase $\Theta$ and reduce the (costly) detection effort so it is immediate that, in optimum, the no bankruptcy condition for the citizen is binding, i.e. $\Theta = 1$. This is in line with Kolm’s [1973] polemic principle, which says that tax evaders should be hung with a probability approaching zero.

If $\rho$ is the belief of the marginal tax evader, then the share of tax evaders $\theta$ is implied by the citizen’s problem (1) as a function in $r$, $\tau$ and $\rho$. The marginal tax payer/evader $i$ is the one for whom $\eta^i$ just offsets the difference $\delta^i = \rho^i\tau + (1 - \rho^i)\tau^e - \rho^i r$, as depicted in figure 2. We assume that tax honesty is uniformly distributed on $(\eta, \bar{\eta})$ with median $\eta^0$, density $h(\eta)$ and c.d.f. $H(\eta) = \int_\eta^\bar{\eta} h d\eta$. In figure 2, we have drawn the function $\eta(i(\theta))$ which assigns agent $i(\theta)$ his tax honesty $\eta(i(\theta))$. $i(\theta)$ is the citizen for whom the share of citizen with lower tax honesty is $\theta$. Putting this otherwise, the mass of citizens $\theta$ who evade taxes for a given $\delta$ is $\theta = H(\eta(i(\theta)))$, where, from lemma 1, $\eta(i) = d^i$. The graph of the function $\eta(i(\theta))$ in figure 2 has a slope of $h$. $h$ can be taken as a proxy for the dispersion of preferences in the society. The share of tax evaders is given by

\[ \theta(\tau, \rho, \tau^e) = 0.5 + \frac{\rho \tau + (1 - \rho)\tau^e - \rho r - \eta^0}{h} \]  

for $h > |A|$, $x = 2(\rho \tau + (1 - \rho)\tau^e - \rho r)$,

$\theta = 1$ for $h < |A|$ and $A > 0$.

\[^9\text{See appendix 1.}\]
\( \theta = 0 \) for \( h < |A| \) and \( A < 0 \).

The marginal effects of the government’s instruments on the share of evaders is \( \frac{\partial \theta}{\partial \tau} = \frac{\rho}{h} \) and \( \frac{\partial \theta}{\partial r} = -\frac{\rho}{h} \), i.e. e the share increases with taxes and decreases with detection efforts. The magnitude of these effects decreases with dispersion. Using the relationships determining aggregate behaviour, the government’s problem is to maximise net revenue from taxing evaders at rate \( r \) and taxing tax payers at rate \( \tau \) above detection costs,

\[
V = \theta(r, \tau, \rho) r + (1 - \theta(r, \tau, \rho)) \tau - C(r, \theta). \tag{3}
\]

We now derive the optimal tax/detection policy-mix for the government:

First, consider the case where \( P = 1 \) and, therefore, \( \rho = 0 \) for all citizens and \( \theta \equiv 0.5 + \frac{\tau}{h} - \eta^0 \)

\[
\frac{\partial V}{\partial r} = \theta - C_r \geq 0
\]

\[
\frac{\partial V}{\partial \tau} = (1 - \theta) \geq 0
\]

In this case, the government increases the tax rate to the point where the restraint \( \tau = 1 \) becomes binding. This is also intutitive: When the reaction to high taxes is low (due to \( \rho \) being small), the government always taxes people as heavily as possible. If \( \theta = 1 \), the tax rate is indeterminate.

Secondly, consider the case where \( 0 < P < 1 \) or \( 1 > \rho > 0 \). Define the Lagrangian \( L = R + \lambda(1 - \tau - s) \), for some \( s \geq 0 \). The FOC’s are

\[
\frac{\partial L}{\partial r} = \theta + (r - \tau) \frac{\partial \theta}{\partial r} - C_r + \frac{\rho}{h} C_\theta = 0 \tag{4}
\]
\[
\frac{\partial L}{\partial \tau} = (1 - \theta) + (r - \tau) \frac{\partial \theta}{\partial \tau} - \frac{\rho}{h} C_\theta = \lambda = 0
\]  
(5)

\[
\frac{\partial L}{\partial \lambda} = 1 - \tau - s = 0 \text{ and } s = 0, \lambda > 0 \text{ or } s > 0 \text{ and } \lambda = 0.
\]  
(6)

(6) are the Kuhn-Tucker conditions which say that either (a) \( \lambda = 0 \) and \( \tau < 1 \) or (b) \( \tau = 1 \) and \( \lambda > 0 \).

Note that \( r \frac{\partial \theta}{\partial \tau} - \tau \frac{\partial \theta}{\partial \tau} = 0 \). Therefore, in the absence of detection costs \( r \) and \( \tau \) are perfect substitutes for collecting taxes. Hence revenues are maximised by setting both instruments equal to one, so government receipts are simply 1. If detection costs are involved, the problem is substantial. From (4) we get an implicit expression for the detection probability:

\[
C_\tau(r^*, \theta) = 1 - \lambda \Rightarrow C_\tau^{-1}(1 - \lambda) = r^*.
\]  
(7)

If the restraint on the tax rate is not binding we have \( \lambda = 0 \). In that case, raising the detection probability by one not only raises receipts by confiscating the income of that part of the population which has chosen to evade taxes, but it also allows to levy increased taxes on the other part of the population. On total, government receipts are increased by 1.

Inserting (7) in (5) yields

\[
(1 - \theta) + (r^* - \tau) \frac{\rho}{h} - \frac{\rho}{h} C_\theta + \lambda = 0
\]  
(8)

If the restraint \( \tau = 1 \) is non binding then \( \lambda = 0 \) and an increase in taxes increases receipts by \( (1 - \theta) \) but also reduces government income due to an increase in tax evaders. This incentive effect depends on the expected impact of the tax (measured by \( \rho \)) and is
proportional to $\frac{1}{h}$. It affects the budget via higher detection costs and according to the relative importance of taxes and fines for the government.

Finally, in an interior solution with $\rho = 1$ we have $C_r = 1$ and

$$\tau^* = r^* + 0.25h - \frac{C_g}{2} + \frac{\eta^0}{2}.$$  \hspace{1cm} (9)

From (9), the tax rate exceeds the detection rate by $0.25h$ minus a correction term for the additional cost of having more tax evaders in the society. The higher $h$ and the greater average pleasure of paying taxes is, the more likely it is that a tax rate of 1 is chosen by the government.

C. Beliefs and aggregate behaviour

Whether or not tax evasion increases when citizens confidence in the permanence of the non constitutional state decreases is not ambiguous. We have:

$$\frac{\partial \theta}{\partial \rho} < 0 \iff t^c > t - r.$$  \hspace{1cm} (10)

With a decrease of $\rho$ the expected cost to the marginal citizen of declaring taxes relative to not declaring taxes shifts from $t - r$ to $t^c$. If the latter were smaller, a decrease in $\rho$ would indeed increase the expected cost of tax evasion. The fact that not only the non constitutional fine is scrapped under the constitution but also the non constitutional share of tax paid hampers the effectiveness of illegitimacy as a device for ensuring sustainability of the constitution. Whereas the former reduces the subjective cost for the citizen of punishing the government, the latter reduces the expected cost for the citizen of complying behaviour,
i.e. of paying taxes. Therefore, the promise that the constitutional government not only promises amnesty but also a correction of the non constitutional tax bill, renders the citizen’s incentives to punishing the government ambiguous.

However, (10) is fulfilled at least in the case where $r > 0$ for some $t^c < 1$. Obviously, due to the special features of taxation, our argument cannot be used to defend any constitutional tax rate. However, it remains true that our argument is the only one in the literature, which suggests that there are constitutional tax rates which are sustainable despite the fact that the government would actually prefer to implement a higher tax rate. In order to proceed in developing our argument, we assume that the constitutional tax rate is sufficiently high, such that (10) is fulfilled:

**Assumption 1** The constitutional tax rate is sufficiently high, such that (10) holds for all $\rho$.

**IV. Tax Evasion in the Different States**

**A. Tax Evasion in the Constitutional State**

In the constitutional state, the citizen knows that fines and taxes are collected with certainty, so $\rho^i = 1$ for all $i$. If the constitutional restraint is binding, the government picks the pre-defined tax rate $\tau^c$ and determines the optimal detection policy satisfies. This satisfies:

$$r(\tau^c) = \frac{h}{2} C_r - \frac{C_0}{2} + r + \frac{\eta^0}{2} \tau^c + 0.25 \eta.$$

Exemplifying the game tree of figure 1, we know that if the government stays with the constitution in the present period, $t$, it does so in the following period $t + 1$. In that case, the government realises the same receipts $V^c$ in both periods.
Example Let the cost function \( C := \frac{3}{2} r \theta^2 + \frac{1}{2} h^2 \) and \( h = 3/2, \eta^0 = 0 \). This implies that both a high effort, \( r \), and a high share of tax evaders, \( \theta \), raise detection costs but that \( \theta \) has this effect only to the extent that the government tries to detect at all. The government is in the constitutional state. The optimal tax-detection policy is \((\tau, r) = (0.8765, 0.7953)\) with net receipts of \( V_t^c = V_{t+1}^c = 0.3858 \). Let the constitutional tax rate be \( \tau^c = 0.6 \). Optimal effort under the restraint is \( r = 0.6 \) and government net receipts are 0.36 (incurred costs of 0.24). A share of \( \theta = 0.5 \) citizens evade taxes.

B. Tax Evasion in the Non Constitutional State

In the non constitutional state, the government policy \((\tau, r)\) is unrestrained in \([0, 1]^2\). \( \theta \) depends on \( \tau, r \), and expectations \( \rho \), so when announcing its policies the government has to take into account the likely reaction of the citizens. If the constitutional tax restraint is actually binding, then the government has an incentive to defect to a higher tax rate \( \tau > \tau^c \), as long as the citizens believe that the government does not switch back, i.e. if \( \rho^i = 1 \) after the defection. Whether or not it wishes to switch back after it has defected, depends on the reaction of the citizens which actually occurs.

If the government switches back in period \( t \), it receives the constitutional income \( V_{t+1}^c = R^c - C^c \) in the following period. In the current period, it resigns enforcing its tax law, i.e. it incurs detection costs of zero, but it also has to go with the constitutional revenue share \( T^nc,c_t = \tau^c(1 - \theta) \), i.e. with what it gets from taxing the actual tax payers at the constitutional tax rate. If it stays in the non constitutional state, it realises its non constitutional income \( V^nc \) in period \( t \) and \( t + 1 \). The government stays in the non constitutional state if the tax revenue foregone by staying does not exceed a certain value \( k \) which expresses the preference
of the government for the non constitutional state.\footnote{A preference of the government the non constitutional state can be made plausible by arguing that it does not have to incur costs for electoral campaigns or the like. Still, our argument does not depend on this assumption, i.e. $k$ might well be negative, but this latter case is less interesting if one wants to explain obedience of the constitution.} $k$ cannot be perfectly observed and at this point we assume that it is independent of past observations of the citizens. The government learns about $k$ after it has announced its tax/detection policy but before it decides on a switch back. This implies that even the fact that the government has already violated the constitution does not contain any information on the true value of $k$. This is clearly a restrictive assumption. In section IV.B. we point out how a dynamic process of learning $k$ can be integrated into the model.

The government switches back if

$$\Delta = V^c_{t+1} + T^{nc,c}_{t} - V^{nc}_{t} - V^{nc}_{t+1} - k > 0. \quad (11)$$

If no switch back is expected on the side of the citizens, i.e. $(1 - \rho^i) = P^i = 0$ for all $i$, the government never switches back:

**Lemma 2** With $P^i = 0 \forall i$ the government stays in the non constitutional state for $k \geq k$ where $k < 0$.

**Proof.** With $P^i = 0$, both, the constitutional and non constitutional government solve the same problem in $\tau$ and $r$. First, $T^{nc,c}(P = 0) < V^c$ because fines and a share of tax income is sacrificed. Then, it suffices to show that $V^{nc}(P = 0) > V^c$. This is evident, because $\tau$ and $r$ are chosen such that they maximize $V^{nc}$ at $\tau^*$ and $r^*$ and $\tau^c \neq \tau^*$. $k < 0$ follows from setting (11) equal to zero with $V^{nc}(P = 0)$ and $T^{nc,c}(P = 0)$. \[\blacksquare\]
Next suppose, that everybody believes in a switch back, i.e. $P^i = 1$ for all $i$. Obviously, there are $k$ for which (11) is fulfilled. Let $\bar{k}$ be the highest $k$, for which this is true (i.e. for which (11) is fulfilled as an equality). This is: $\bar{k} = V^c + T^{nc,c}(P = 1) - 2V^{nc}(P = 1)$.

Then the switch back mechanism is effective in this model, if $\bar{k} > \underline{k}$, which amounts to saying that with stronger beliefs in the non permanence of the non constitutional state, the government has to be more stubborn to stay in the non constitutional state. Demanding $\bar{k} > \underline{k}$ is equivalent to the statement

$$T^{nc,c}(P = 0) - T^{nc,c}(P = 1) < 2V^{nc}(P = 0) - 2V^{nc}(P = 1).$$

This is, as $P$ changes from 0 to 1, the implied decrease of non constitutional income, $V^{nc}$, must be bigger than the reduction in switch back income, $T^{nc,c}$, which is due to the fact, that tax evasion increases with $P$.

We assume that (12) holds. Observing that in (12) $T^{nc,c}(P) = \tau^c(1 - \theta(P, \tau, r))$ and $V^{nc}(P) = \tau(1 - \theta(P, \tau, r)) + r\theta(P, \tau, r) - C(r, \theta(P, \tau, r))$ and taking derivatives with respect to $\theta$, it is easy to see that a sufficient condition for this to be true is

$$2(\tau - r + C_\theta) > \tau^c$$

for all $\theta_{(\tau, r, P)}$. 

A higher $\tau^c$ is more difficult to sustain because the reduction in switch back income following a rise in $P$ is stronger, the higher is $\tau^c$. The reason why we require a slightly stronger condition than necessary is that (13) implies

$$\frac{\partial \Delta}{\partial \theta} > 0$$

\footnote{Note that $\tau - r + \frac{\eta^0}{2} + C_\theta/2 = \tau^c$.}
which we use in the global game. This is, an increase in tax evasion makes it more likely that the government switches back. Note that assumption 1 implies that a rise in $P$ increases $\theta$ and by (13) this transforms into a higher probability of defection ($P$). In part 2 of the appendix we show, that condition 13 is fulfilled, if

**Assumption 2** (a) $\tau^c < 2h(1 - \theta)$ if $\tau < 1$ and (b) $\tau^c < 2(C_\theta + 1 - r^\ast)$ if $\tau = 1$.

This condition is derived from the first order condition of the government. Without the effect on switch back income, the r.h.s. in (13) would be zero an (12) would be always fulfilled. To put thing into context, it is again the fact that the constitutional government taxes income declared under the non constitutional regime why we need a separate condition to ensure that the illegitimacy argument works. We have the surprising effect that tax evasion increased the pressure on the government to return to the constitution when the constitutional tax rate is sufficiently low. Note, that this is isolated from the question of whether a higher or lower level of $k$ is compatible with constitutional stability. With assumption 2 satisfied, we immediately get

**Lemma 3** $\overline{k} > \underline{k}$.

**Proof.** Under assumption 2 (13) is true which implies (12).

In our example, (11) is fulfilled even for $k = 0$, so from lemma 2 we immediately have $\overline{k} > \underline{k}$.

**Example (continued, a)** If all citizens have $\rho = 0$, then the best government policy is to set the tax rate at $t = 1$ and detection efforts $r = .36$. The share of evaders, $\theta = .9$, receipts are $.424$ and costs $.2592$. Net receipts are, therefore $V_{t}^{nc} = V_{t+1}^{nc} = .1648$. If
the government is content to go with the constitutional share of its tax receipts, it has a switching pay off of $T_i^{nc,c} = .06$. Setting $k = 0$, the criterion (11) now reads

$$T_i^{nc,c} + V_{t+1}^{c} > V_t^{nc} + V_{t+1}^{nc}.$$  

In this case, this is true because $.42 > .3296$.

We can turn this argument and say, that there is some $k > \overline{k}$ which makes the non bracketed part of (11) negative, such that no-one evades taxes for the reason of an expected switch back (i.e. the share of tax evaders which induces the government to switch would have to be larger than 1 which is clearly impossible) and tax evasion is governed by preferences for tax honesty alone.$^{12}$ On the other hand, there is $k < \underline{k}$ such that everybody sets $P = 0$, in which case no-one pays taxes.

**Proposition 1** *In the case of certainty, if $k \in (\underline{k}, \overline{k})$, there are two equilibria: one, where no-one declares income for tax purposes and one, where everybody declares income for tax purposes.*

**Proof.** With certainty, either $P = 1$ or $P = 0$. For all $k \in (\underline{k}, \overline{k})$ (11) is fulfilled if $P = 0$ (from lemma 2) or violated if $P = 1$ (from assumption 2). $\blacksquare$

Even though assumption 2 is sufficient to insure the existence of a trigger mechanism in the global game which we discuss in the following section, we are going to show that in our example even a common belief of $P = 0.5$ is sufficient to force the government back into the constitutional state. The propositions derived in this paper are much more powerful than this example in that they hold quite generally provided that assumption 2 is fulfilled.

$^{12}$It is exactly the event that citizens could make the government switch without taking account of the fundamental parameters of the model which has to be excluded in order to solve the problem of multiplicity of equilibria.
However, the fact that there are parameter constellations, where the mechanism works even with $k = 0$ and $P = 0.5$ certainly gives the model additional appeal.

**Example (continued, b)** Finally, if every citizen holds $\rho = 0.5$ the optimal policy of the government is to levy a tax rate of $\tau = 1$ and choose effort $r = 0.7208$ in the non constitutional state. A share of $\theta = 0.7931$ choose to evade taxes and net government receipts are $0.2165$. With a constitutional tax rate of 0.6, the switch back pay off is $T_{1}^{nc,c} = 0.1242$. This again satisfies criterion (11) with $k = 0$, because $0.4842 > 0.423$.

V. The Game with Imperfect Information of $k$

In this part of the paper we derive a unique equilibrium for the game between the government and the citizens, using the results on global games by Carlsson/van Damme (1993) and Morris/Shin (1998, 1999, 2000).

A. The switch-back condition

Proposition 1 suggests that the game has multiple equilibria under certainty. In a global game, introducing uncertainty about the true value of $k$ removes multiplicity of equilibria: If the citizens observe only a noisy signal of $k$, then there is a unique value $k^{*}$, such that (1) enough citizens receive a message exceeding a critical signal level such that reacting upon that signal level would force the government back into the constitution. (2) Reacting (i.e. evading) when the signal exceeds the critical level is a strategy which survives iterated elimination of dominated strategies.

From (14) and because $\frac{3k}{1k} < 0$ there exists a strictly increasing function $\phi(k)$ for $k \in (k, \bar{k})$ where upon increasing $k$, $\Delta$ changes from a positive to a negative value for a given share of
evaders $s$. We have depicted $\phi(k)$, which gives the critical mass of evaders $\theta$ as a function of $k$ in figure 3. To determine $\phi(k)$ we have to insert the mass of evaders for $P = 0$, $\theta$. For higher $k$, no mass of evaders which can be generated by this model is sufficient to elicit a switch back. Likewise, we have $\phi(k) = \theta$. Recall that from assumption 1, $\theta < \overline{\theta}$.

insert figure 3 about here

B. The strategies of the citizens

If the true state is $k$, a citizen observes a signal $x^i$ which is the realisation of a random variable distributed uniformly on the interval $[k - \varepsilon, k + \varepsilon]$, with $\varepsilon$ being small. Consider the utility of a citizen from the action "tax evasion", conditional on his subjective probability that a switch back takes place, $P(x^i)$:

$$u^i \equiv U^i(y^i = 0) - U^i(y^i = 1) = (1 - P(x^i))(\tau - r) + P(x^i)\tau^c - \eta^i. \quad (15)$$

$\tau - r - \eta^i$ is his utility loss compared to declaring if he evades taxes and the switch back does not take place in which case the non constitutional policy is implemented. If $\tau - r - \eta^i \geq 0$, he would have evaded taxes in anyway, so this citizen does not add to the mass which would eventually trigger the switch back. The marginal value for which this is true is $\underline{\eta}$. If $\underline{\eta} < \overline{\eta}$ then some citizens are stubborn evaders and $\theta = H(\underline{\eta}) > 0$. $\tau^c - \eta^i$ is the utility from evading taxes, i.e. the money saved, if the switch back takes place and non constitutional taxes and fines are sacked. Again, there is a value $\overline{\eta}$ such that $\tau^c - \overline{\eta} = 0$. If $\overline{\eta} > \underline{\eta}$ some citizens always pay taxes and $\overline{\theta} = H(\overline{\eta}) < 1$. It is easy to see that assumption
1 implies\(^\text{13}\) \(\frac{\partial u^i}{\partial P} > 0\) and that \(\bar{\eta} > \underline{\eta}\). Furthermore, there exist values \(\bar{\eta} \in (\tau - r, \tau^c)\) such that (15) matters for the decisions of the citizens if \(h \to 0\) and tax honesty of all citizens converges to \(\bar{\eta} = \eta^0\), i.e. \(\underline{\bar{\eta}} = \bar{\eta} = \frac{\eta}{2}\). In this case, the evasion decision is governed by \(P(x^i)\) alone.

Next we show that the actual share of tax evaders \(s\) in figure 3 is a function of the true state \(k\). Each citizen applies a switching strategy: evade if \(x^i < z^i\). Now consider a citizen 0 with \(\eta^0\) who has received a signal \(x^0\). It is straightforward to derive equilibrium behaviour for the case of homogenous citizens (i.e. \(\eta = \bar{\eta}\) for all citizens). After observing \(x^0\), a citizen can infer the range of \(k\) and think about the signals which other citizens have received. The expected value \(E_k|x^0\) is \(x^0\) itself and \(k\) is distributed on \((x^0 - \varepsilon, x^0 + \varepsilon)\). From this we can induce the probability \(\psi(q, z^i|x^0)\) which a agent who has received a signal \(x^0\) assigns to the event that the share of players who receive a signal \(x^i\) falling short of \(z^i\) is at least \(q\). If the true state is \(k\), the share of citizens observing a signal falling short of \(z^i\) is given by the c.d.f. \(W(z^i|k) = \frac{z^i - k + \varepsilon}{2\varepsilon}\). Say that \(W(z^i|k) = q\). Then at least \(q\) citizens observe a signal no greater than \(z^i\), if \(k = z^i - 2\varepsilon q + \varepsilon\). To find the probability of at least \(q\) citizens having observed \(z^i\) or smaller we have to pick those \(k\) which are consistent with this event from all \(k\) which are possible from the observation \(x^0\):

\[
\psi(q, z^i|x^0) = \int_{x^0 - \varepsilon}^{z^i - 2\varepsilon q + \varepsilon} \frac{1}{2\varepsilon} \, dk = \frac{z^i - x^0}{2\varepsilon} + (1 - q). \tag{16}
\]

\(\psi\) is decreasing in \(x^0\), i.e. the probability attached to a switch back decreases with the signal. The critical observation is \(\xi = x^0 = z^i\), which yields \(\psi^*(q, \xi|\xi) = (1 - q)\). There is a unique value \(P^* = \psi^*(\phi(k), \xi|\xi)\) such that \(P^*\) satisfies condition (15) for the marginal

\(^{13}\)With (14) another implication is that decisions to evade are strategic complements.
citizen, i.e. \((1 - P(\phi(k), \xi|\xi))(\tau - r) + P(\phi(k), \xi|\xi)\tau^e = \bar{\eta}.\) From (16), \(P\) is given by \((1 - q).\) Given that \(u\) is increasing in \(P,\) the value \(q\) satisfying (15) is unique. This is sufficient to establish the equilibrium. For completeness, we derive the critical signal. At \(k^*\) it must be the case that the share of tax evaders \(s(k) = \phi(k).\) If \(k^*\) is the true value, then at \(k^*\) it must be that \(W(\xi|k) = s(k).\) Because \(\phi(k)\) is increasing and \(W(\xi|k)\) is decreasing in \(k,\) the value \(k^*\) satisfying this condition is unique. Finally, this implies

\textbf{Proposition 2} The global game has a unique equilibrium point \(k^*\) with \(\underline{k} < k^* < \overline{k}\) such that the government switches back if \(k < k^*.\)

\textbf{Proof.} The existence and uniqueness of \(k^*\) is established above. Showing that \(k^* > \underline{k}\) implies showing that at \(\underline{k}\) we have \(s(k) > \phi(k).\) This to be an equilibrium, we need \(\phi(k) = s(k) = \bar{\theta},\) so \(\psi^* = (1 - \bar{\theta}).\) But in fact from the definition of \(\underline{k},\) \(\psi = 0\) is sufficient to cause \(\bar{\theta}\) citizens evading taxes, so \(s(\underline{k}) > \bar{\theta}.\) Iterative elimination of dominated strategies establishes \(k^*\) (see Morris/Shin (2000). A similar argument applies to \(k^* < \overline{k}\).

The more complicated case arises with heterogenous citizens. In this case we need to simultaneously find a distribution of cut off points for the citizens together with the unique point \(k^*\) such that the decision of each citizen on tax evasion given his signal is in accordance with (15) when the probability that \(\phi(k^*)\) citizens have observed a signal falling short of their cut off point is determined by the common distribution of the individual cut off points, \(\xi(\eta),\) and the signal generated by \(k^*, x.\) Pech (2001) proofs that \(k^*\) is unique whenever the distribution \(f(\xi)\) is unique and shows the existence of a unique solution \(f(\xi)\) for the special case in which (15) holds with \(P = 0.5\) for the median citizen.

\textsuperscript{14}Note that \(\xi = k^*\) only if \(q = 0.5.\)
VI. DISCUSSION AND EXTENSIONS

A. Dispersed societies

Dispersion of the society is represented by the parameter $h$ which governs the distribution of the $\eta^i$'s. A higher $h$ implies more extreme behavior at the opposite ends of the scale. $h$ has an influence on the mass of citizens $(\bar{\eta} - \underline{\eta})$ who respond to a change in $P_i$. The condition in assumption 2, however is weakened if $h$ increases as long as $h \frac{d(1 - \eta)}{dh} + (1 - \underline{\eta}) > 0$ thus making higher constitutional tax rates sustainable without violating $k < \bar{k}$. On the other hand, there is a counter balancing effect in the condition of assumption 2, leaving the net effect uncertain. On the other hand, a rise in $h$ raises $\tau - r$ which makes the non constitutional state more attractive for the government and it becomes more likely that the government defects on the constitution and does not switch back.

The results on the global game are not affected by $h$ as long as assumption 2 holds. However, if a society is composed exclusively of citizens for whom either $\eta < \underline{\eta}$ or $\eta > \bar{\eta}$, the trigger mechanism fails. In such a society, the government would set both $\tau$ and $r$ equal to one, thereby taxing the voluntary tax payers and the stubborn tax evaders at a maximal rate. In this sense, lower scale dispersion is a defence against unconstitutional usurpation of power.

B. The decision to defect from the constitution

Up to this point we have considered the case of a government which had already defected from the constitution. We have shown that depending on the realisation of $k$, the government is forced to switch back by the reaction of the citizens. And, by establishing $k^* > \bar{k}$ in
propoition 2 we have an argument that the reaction of the citizens actually matters: $k$ is the critical parameter under which the government switches back if the citizens assume, that the law is actually going to be enforced. Yet, what we were ultimately looking for was an argument that would deter the government from defecting at the outset. This is done in this section.

Here we model the stage in period $t - 1$ when the government is in the constitutional state. The taste of the government at this stage is $k_{t-1}$ and $k_{t-1}$ is common knowledge. This is important, because in this case the decision of the government on whether to defect or not does not reveal anything to the citizen and is not subject to strategic considerations.

The government has to decide on whether it defects from the constitution, in which case it is in the non constitutional stage in period $t$ and faces the game as in figure 1, or whether it stays with the constitution, in which case it receives an income of $V^c$ in period $t$. Our only restriction on the timing remains that the government can choose the constitution only by announcing a constitutional tax rate - in which case it is restricted carry out taxation accordingly. For the decision in period $t - 1$ this means that it may as well defect one period later. So its decision criterion becomes

$$V^c \geq pT^{nc,c} + (1 - p) \int_{k \geq k^{**}} \frac{w(k|k_{t-1})}{1 - W(k^{**}|k_{t-1})} V^{nc}(\tilde{P}(k)) dk + k_{t-1}. \quad (17)$$

The left hand side of this inequality is the expected pay off the government receives in $t$ when it slips into the non constitutional state. $p$ is the probability that $k < k^{**}$ with $k^{**}$ being the switching point in the game with prior information on $k_{t-1}$. In the case, where $k > k^{**}$, the government switches back and receives $T^{nc,c}$. In the other case, it stays
in the non constitutional state. For any \( k \geq k^* \), the state of the government induces a probability distribution on the side of the citizens, \( \tilde{P}(k) \). Note, that whenever \( k^* > k \), the government stays outside the constitution with certainty. However, because citizens only receive a signal \( x \sim (k - \varepsilon, k + \varepsilon) \), some citizens (but at most \( \phi(k^*) \)) do (erroneously) evade taxes. In order to prove that the trigger mechanism is substantial, it is sufficient to note that \( V^{nc}(\tilde{P}(k)) > V^{nc}(P = 0) \) for all \( k \geq k^* \). The term involving \( w \) is the probability distribution of \( k \), conditional on \( k_{t-1} \) and \( k \geq k^* \).

For the government to be able to make a decision in \( t - 1 \), it has to know \( p \) in relation to \( k_{t-1} \), which is the stochastic process, which determines \( k \). We assume that \( k \) results from \( k_{t-1} \) according to \( k \sim \frac{1}{2E} \) for \( k \in (k_{t-1} - E, k_{t-1} + E) \). As before, for any realisation of \( k \), the distribution of signals to the citizens is equally distributed on \( (k - \varepsilon, k + \varepsilon) \). With this set up, we have a process, through which the citizens are indeed able to update their beliefs of \( k \), when they receive the signal \( x \). However, it turns out that allowing for such an updating process in \( t \) gives rise to multiplicity of equilibria (see appendix 3). We, therefore, assume that \( \varepsilon \rightarrow 0,^{15} \) in which case \( k^*|k_{t-1} = k^* \) for all \( k_{t-1} \in (k^* - E, k^* + E) \) where we assume that \( (k^* - E, k^* + E) \subset (\underline{k}, \overline{k}) \). In that case, \( p = (k^*|k_{t-1}) = \frac{k^* - k_{t-1} + E}{2E} \). Therefore, \( p \) increases from 0 to 1 as \( k_{t-1} \) goes from \( k^* + E \) to \( k^* - E \). It can be shown that the r.h.s. of (17) decreases in \( p \). Furthermore we have \( T^{nc,c} < V^c \) and \( V^{nc}(\tilde{P}(k^* + E)) > V^{nc}(P = 0) > V^c \). Therefore there is a unique switching point \( k_{t-1} = k^*_{t-1} \) at which (17) holds as an equality.

It is straightforward to show, that the switch back mechanism economically matters. Observe, that without the switch back mechanism, the condition for the government to

\(^{15}\)Morris/Shin (1998b) have to make a similar assumption to insure that \( \psi' < 0 \) in a model with normally distributed signals and a Brownian motion stochastic process.
defect on the constitution would read

\[ V^c \geq V^{nc}(P = 0) + k_{t-1}. \]  (18)

Let \( k^0_{t-1} \) be the limiting value for which this inequality becomes binding. Now suppose that \( k^{**}_{t-1} = k^0_{t-1} \). As we have shown, \( p > 0 \) at \( k^{**}_{t-1} \). Clearly, \( T^{nc,c} < V^{nc}(P = 0) \). Furthermore, \( V^{nc}(\bar{P}(k)) < V^{nc}(P = 0) \) for all \( k \geq k^* \). Therefore, if (18) is binding for \( k^0_{t-1} \), (17) is not binding. Therefore, \( k^{**}_{t-1} > k^0_{t-1} \) and we have the following proposition:

**Proposition 3** For \( k_{t-1} \in (k^* - E, k^* + E) \) the switch back mechanism is substantial: The critical value from, which on government defects without the switch back mechanism, \( k^0_{t-1} \), is smaller than \( k^{**}_{t-1} \).

**Proof.** See discussion above. ■

VII. CONCLUSION

In this paper we have shown how a government who is otherwise inclined to defect from the constitution is forced to obey by the difficulties of law enforcement it faces after the violation. Such difficulties result because a constitution promises to leave citizens who violate a non constitutional law without punishment, so the expectation that after a defection a voluntary switch back to the constitution might take place ultimately becomes self fulfilling. We have established that the switch back mechanism is substantial in that it puts more pressure on the government to stay in the constitutional state when compared to a world where the switch back mechanism is not active. From a policy perspective, this means that the fact that the constitution establishes a state under the rule of law lends additional stability to
the constitution when it is threatened by the government. This is an important finding, because as we see it, it is the only constitutional principle which has this inherent deterrence power. In a technical sense, we have simply exploited the fact, that this principle is capable of breaking the chain of opportunistic behaviour. In that sense this paper is in a tradition which argues that dynamic policy restraints do matter.

We believe that the idea on which this paper is based should lend itself to extensions in various directions. One way could be to study the capital taxation problem where the citizens would act in an explicit dynamic framework. Another route that future research might take is the application of the illegitimacy issue in various other fields of the judicial science.

VIII. Appendix

A. Appendix 1

Using $\Theta = 1$ in (1) the citizen declares taxes (i.e. set $y^i = 1$) if $\rho^i \tau + (1 - \rho^i)\tau^c - \eta^i < \rho^i r$ so for the marginal citizen it must be that

$$\eta = \rho(\tau - r) + (1 - \rho)\tau^c.$$ 

As this is the equation for the marginal citizen, it must be that

$$\theta = H(i|\eta^i \eta) = \int_\eta^\delta \frac{1}{s-\eta} d\eta = \int_\delta^\infty S \, d\eta. \text{ The distribution of } \eta(\theta) \text{ is}$$

$$\eta(\theta) = h(\theta - 0.5) + \eta^0.$$ 

From this, we have $\theta = 0.5 + \frac{\eta(\theta) - \eta^0}{\eta}$, which together with $\eta(\theta) = \eta$ and the restraint $\theta \in [0,1]$ yields (2).
B. Appendix 2

From the f.o.c. we get
\[ \frac{h}{\rho}(1 - \theta(\rho, .)) + \lambda = (\tau - r) + C_\theta \]

(a) Consider \( \lambda = 0, \ tau < 1. \) Obviously, (13) is fulfilled if
\[ \frac{h}{\rho}(1 - \theta(\rho, .)) > \frac{1}{2} \tau_c. \]

Observing that \( \theta(\rho = 1) = \underline{\theta} \) and \( \theta(\rho = 0) = \overline{\theta}, \) we find that
\[ \lim_{\rho \to 1} \frac{h}{\rho}(1 - \theta(\rho, .)) = h(1 - \theta) \]

\[ \lim_{\rho \to 0} \frac{h}{\rho}(1 - \theta(\rho, .)) = -h\theta_\rho \text{ if } \overline{\theta} = 1 \]

\[ \lim_{\rho \to 0} \frac{h}{\rho}(1 - \theta(\rho, .)) \to \infty \text{ if } \overline{\theta} < 1 \]

where we have applied l’Hopital’s rule to get the second limit. Noting that \( \theta_\rho(\rho = 0) = - (\overline{\theta} - \underline{\theta}), \) the second line is more binding than the first. The third line is less binding than the other two. Therefore, it is sufficient for the condition \( (\tau - r) + C_\theta > \frac{1}{2} \tau_c \) to be fulfilled that \( h(\overline{\theta} - \underline{\theta}) > \frac{1}{2} \tau_c. \)

(b) Consider \( \tau = 1, \lambda > 0. \) Using \( \tau = 1 \) and \( r^* \) in (13) yields \( C_\theta + 1 - r^* > \frac{1}{2} \tau_c. \)

C. Appendix 3

In this appendix, we show how a prior \( k_{t-1} \) affects the equilibrium value \( k^{*'} \) at the next stage.

(a) If \( x^0 \in (k_{t-1} - E + \varepsilon, k_{t-1} + E - \varepsilon) \) we have \( w(k|x^0, k_{t-1}) = w(k|x^0) = \frac{1}{2\varepsilon} \) for \( k \in (x^0 - \varepsilon, x^0 + \varepsilon). \) In this case, set \( X = [x^0 - \varepsilon, x^0 + \varepsilon] \) of \( k \) which is possible from the
observation \( x^0 \) is covered by the set \( K = [k_{t-1} - E, k_{t-1} + E] \) of \( k \) which is possible from the prior \( k_{t-1} \), so the prior does not affect the probability distribution over \( k \).

(b) If \( x^0 \in (k_{t-1} - E, k_{t-1} - E + \varepsilon) \) we have \( w(k|x^0, k_{t-1}) = \frac{1}{2\varepsilon - D} \) for \( k \in (\bar{x} - (\varepsilon - D/2), \bar{x} + (\varepsilon - D/2)) \). In this case it is clear that \( k \) is in the intersection \( X \cap K \). The length of the interval for the posterior distribution of \( k \) is shortened by the absolute difference \( D = |X| - |X \cap K| \) where \( D = |k_{t-1} - x^0 + \varepsilon - E| \). The updated expected value for \( k \) is \( \bar{x} = \frac{k_{t-1} - E + x^0 + \varepsilon}{2} \). (16) becomes

\[
\psi(q, z^i|x^0, k_{t-1}) = \int_{\bar{x} - (\varepsilon - D/2)}^{z^i - 2\varepsilon + \varepsilon} \frac{1}{2\varepsilon - D} \, dk.
\]

Using the identity \( k_{t-1} = x^0 - \varepsilon + E + D \) for \( x^0 < k_{t-1} \) in the expression for \( \bar{x} \), and solving the integral we find

\[
\psi(q, z^i|x^0, k_{t-1}) = \frac{z^i - x^0}{2\varepsilon - D} + (1 - q)\left(\frac{2\varepsilon}{2\varepsilon - D}\right) - \frac{D + E - \varepsilon}{2\varepsilon - D}.
\]

Note that \( E \) and \( \varepsilon \) in the last term are just correcting for the same values in the expression for \( D \). Basically, we want to have that \( \psi \) decreases with \( x^0 \) which was satisfied in the case without a prior (so the lower the signal, the more certain citizens are that a switch back will take place). Now, the effect of \( x^0 \) in the first and last term cancel out and the expression in the denominator dominates but we still have \( \frac{\partial \psi}{\partial x^0} < 0 \).

However, if we solve for the critical observation \( z^i = x^0 = \xi \)

\[
\psi^\star(q, \xi|\xi, k_{t-1}) = (1 - q)\left(\frac{2\varepsilon}{2\varepsilon - D}\right) - \frac{D + E - \varepsilon}{2\varepsilon - D}.
\]

Obviously, the expression for \( \psi^\star \) is not monotonic in \( \xi \). The intuition for this finding is as follows. For any single observer, having observed a higher \( x^0 \) makes it more likely that other
people have observed a lower than their (given) threshold values. If we look at the critical observation (involving $\psi^*$), moving my observation to the right and the threshold values of everybody else at the same time, does not make any difference. However, in the case of a prior which is binding for the updating process, my guess of the distribution of $k$ moves only sluggishly with a change in my observation. For the critical observation that means when everybody moves his threshold point to the right, my probability conditional on my (critical observation), that they receive a signal below their threshold actually decreases. On the other hand, for a given expected value $\overline{x}$, a lower $x^0$ makes the distribution of $k$ more narrow or raises $D$. In the limit, this effect dominates, because as $D$ goes to $2\varepsilon$, there is absolute certainty about the true value of $k$.

Non monotonicity of $\psi^*$ in $\xi$ may lead to non uniqueness: For this we check wether a value $k'$ can be supported as an equilibrium value. This is the case if at $k'$ we have $\phi(k') = s(k') = q$ where $\psi^*(q, \xi, k_{t-1})$ solves (15). From non monotonicity there can be another value $\xi^1 > \xi^0$ such that $\psi^*(q, \xi^1, k_{t-1}) = \psi^*(q, \xi^0, k_{t-1})$. Therefore, a citizen having received $x = \xi^0$ may conclude that $q$ citizens have received a signal lower than $\xi^0$ with probability $\psi^*$ if the true value is $k^0$. But if some of these citizens coordinate at $\xi^1$ (with corresponding $k^1 > k^0$), then $s|k^0 < q$. On the other hand, if people receive the higher value $x = \xi^1$ but others coordinate at $\xi^0$, then $s|k^1 > q$, contradicting that $k'$ is an equilibrium.

(c) Finally consider the case where $x^0 \in (k_{t-1} + E - \varepsilon, k_{t-1} + E)$. Now again we have $w(k|x^0, k_{t-1}) = \frac{1}{2\varepsilon - D}$ for $k \in (\overline{x} - (\varepsilon - D/2), \overline{x} + (\varepsilon - D/2)$ yielding probability distributions $\psi(q, z^i|x^0, k_{t-1}) = \frac{z^i - x^0}{2\varepsilon - D} + (1 - q)(\frac{2\varepsilon}{2\varepsilon - D})$.

which eventually increases in $x^0$ (because for $D \to 2\varepsilon$ we have $\psi \to \infty$ so $\psi = 1$ is reached
for greater values of $D$). In this case, citizens cannot coordinate at a threshold value at all, because a higher value of $k$ carries a higher signal and induces more people to believe that others hit their threshold value.

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**References**


Figure 1: The game between government and citizens

government decides whether to execute r or to correct tax law

government learns k

citizen declare taxes (i.e. announce θ)

government announces τ and r
Evasion in response to fiscal parameters
Figure 3: Actual and critical mass of tax evaders and the switching point $k^*$