Penalties in the theory of equilibrium tax evasion: Solving King John's problem*

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Abstract

We characterize equilibria of an income reporting game with bounded returns and no commitment where detected tax evaders are charged the maximally feasible amount. Introducing partial commitment to punishment relief eliminates multiplicity of equilibria. We identify a unique limit equilibrium where the poorest citizens evade, intermediate citizens are honest and the richest citizens are indifferent between evading and truth-telling. For small tax rates and auditing cost, committing to a discretionary punishment relief scheme increases expected tax revenue.

Keywords: Tax evasion, signaling, optimal punishment, JEL codes: D82, H26, K42

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1 Introduction

It is a fundamental insight, received from the standard analysis of tax evasion, that efficiency calls for tax evaders to be charged the maximally feasible fine. A sufficiently high fine even makes it possible to implement the first best solution. Kolm (1973), taking up an argument introduced by Becker (1968), famously claimed that efficiency considerations in a world with costly detection would require tax evaders to be hung with a probability approaching zero. The intuition is that a given deterrence effect can be achieved at lower cost by decreasing detection efforts and increasing severity of punishment.[1] In order to explain why historically the severity of punishment for crimes has been declining since the middle ages while the cost of enforcement has been increasing, economists have to resort to the argument that the general acceptability of severe forms of punishment has decreased.[2] But, hypothetically speaking, even if we were to reject the idea that tax evaders should be hanged, would it not make sense to make them liable with their whole belongings to vouch for the correctness of their tax declarations?

In this paper we show that even if the government follows a narrowly defined objective of maximizing its tax revenue, it may find it worthwhile to charge tax evaders less than the maximally feasible amount. We reconsider the question of optimal punishments in an equilibrium framework where the enforcement agency itself adjusts its behavior to the actions of the citizens.[3] For this we apply the equilibrium tax evasion model of Landsberger/Monderer/Talmor (2000) - henceforth LMT - to a predatory setting where the maximally feasible monetary punishment is imposed on tax evaders. We characterize equilibria of the income reporting game. We show that introducing the possibility for the government of granting a grace value eliminates multiplicity of equilibria. Moreover, we show that for a small tax rate and small auditing cost the possibility to commit to a discretionary punishment relief scheme increases expected tax revenue.

[1]See Borck (2004), Boadway/Richter (2005) and Dittmann (2006) for a discussion of the relative merits of different fine regimes in the standard model. Theoretical consequences of different fine regimes in the economics of crime are discussed in Polinsky/Shavell (2000).

[2]Dami/al-Nowaihi (2006) give an overview of approaches to solve this issue. They argue that if citizen behavior is explained by prospect rather than expected utility theory less severe punishment can be optimal.

[3]Pyne (2004) shows that with behavioral adjustments by police officers to legal standards, making it harder to convict criminals can reduce crime.

In order to appreciate our approach, imagine King John setting the tax rate for the whole of his lands but delegating the task of enforcing the tax rule to local sheriffs.[4] The sheriff hires tax inspectors as subcontractors on the promise that they may keep anything that falls into their hands if a village they inspect is found to have paid less than the proper tax. Here, the inspectors - enforcement agencies essentially - impose the maximum fine as the villagers vouch with all their belongings for even a small breach of the tax code. Would it be desirable for King John to restrain his enforcers and impose less than the maximum fine on at least some undutiful villages?

In an equilibrium model of tax evasion it is not immediately clear what the answer to this question is.[5] If the fine is lowered compared to the predatory equilibrium where undutiful villages are looted, the decreasing reward to the enforcers induces a reduction of their efforts. Just like in the standard model of tax evasion, villagers respond by increasing tax evasion. A full account of equilibrium behavior, however, has to allow for a changing composition of evading villages. The unique limit predatory equilibrium is characterized by the poorest villages underreporting, intermediate villages acting strictly honestly while the richest village is indifferent between these two options. In those circumstances it could pay for the King John to be more lenient with an intermediate village which hands in a false report if it could thereby persuade the richest village not to hand in precisely the same false report. Here, in equilibrium, the former reports will drive out the latter. We can show that if the intermediate villages' total post tax income which can be appropriated by the tax inspectors does not exceed the total post tax income of the richest villages there generally exists a stable revenue superior equilibrium with discretionary punishment relief.[6]

As we point out in our discussion, our results generalize to settings where the sheriff acts as a profit maximizer and do not hinge on the assumption

[4] We refer here, of course, to the mythological villain also known as Prince John. That the historical figure, King John, signed the magna carta which is the first document to formally constrain the powers of government, adds a nice twist to our story.

[5] Note that in an Nash equilibrium zero tax evasion cannot be supported except in trivial cases: It cannot be an equilibrium for no-one to submit a false report because the enforcement agency would not want to audit any income report. But if no income reports are audited every citizen would want to deviate and submit a false income report with positive probability.

[6] In a modern variant of this tale, German minister of finance Steinbrueck has, after some consideration, decided to subject citizens availing of the Swiss tax haven to less than the maximally feasible scrutiny.

that revenue from tax evasion does not accrue to the king. The reason why we choose our particular motivating story is that it naturally suggests non commitment as a behavioral standard whilst with a monolithic tax administration there might be conflicts between attempts to commit to more rigorous tax enforcement in the short term and adaptive behavior in the long term. It is a remarkable result of our analysis that making a certain degree of commitment available in a Nash equilibrium turns behavioral standards suboptimal which are obtained as optimal standards in a model with full commitment.

Equilibrium models of tax evasion have been pioneered by Reinganum and Wilde (1986) and LMT. Alm and McKee (2004) report experimental results on coordination games where the tax authorities select their auditing strategy endogenously. Reinganum/Wilde construct a separating equilibrium where each citizen underreports and the enforcement agency can correctly infer their type. However, due to increasing auditing costs only a fraction of reports in each income category is actually audited. In the equilibrium of Reinganum/Wilde the government applies an audit probability which decreases in reported income. LMT focus on a model with an exogenously given, linear fine on evaded tax, linear auditing costs and risk neutral citizen who may use randomized reporting strategies. In equilibrium, income reports are audited up to a threshold level, citizens with income above the threshold report just more than the threshold and citizens below the threshold randomize over all income reports and are indifferent to truth telling. All reported income levels which get audited are audited with the same probability.

In our model, by contrast, for sufficiently low detection costs, all citizens with incomes above a threshold strictly prefer evading taxes over truth telling but randomize over the report which they submit. The income threshold itself is the highest income reported - and audited - in equilibrium. Citizens below the threshold income are either honest or report zero income. The existence of a threshold income level can be easily understood: In the predatory situation citizens end up with the same (zero) income when detected. Because for any false income report, a richer individual saves more taxes than a poor individual, a rich individual is always more easily persuaded to submit any given false report. That the zero income report is potentially attractive for every income group is a consequence of the linear specification of the model. An agent is indifferent between a lottery in which she looses everything with probability r' and having her income taxed at rate r'. As a consequence, the income reporting game admits multiple equilibria, each of which is supported by a detection probability which decreases in reported income. a unique equilibrium, however, obtains if there is a vanishingly small "grace" value which is left to detected tax evaders. Such a grace value has relatively more worth to poorer villagers, so they tend to be more easily tempted to submit a false report. The only equilibrium surviving this refinement is one where the richest village is indifferent between truth-telling and evasion and the poorest evade. Moreover, we demonstrate that if the king can commit to a discriminatory fine scheme which depends on the tax evaders' true and reported income, tax revenue may increase. In particular, for sufficiently small tax and detection cost, stable, revenue superior equilibria with commitment generally exist.

Section 2 sets out the model. Section 3 derives and refines the predatory equilibrium. Section 4 discusses the substantive effects of a commitment possibility. Section 5 concludes.

2 The model

Income y is a realization of an integer in $Y = \{y^0, .., y^N\}$ with $y^0 = 0$.[7] In the (finite) population, the probability that an agent is of type y^k is n^k. We assume for every k, $n^k > 0$. We consider a two stage signalling game where in the first stage of the model citizens declare their income for tax purposes. A citizen of income class y^k reports income $x^i \in X^k$ where $X^k = \{y^0, .., y^k\}$. That is, an income in Y may be reported and no citizen reports more than her true income. There is a proportional tax rate on declared income $\tau \in (0, 1)$. In the second stage, each tax enforcer is assigned a class of income reports, x^i, for which he is in sole charge[8] and he subsequently selects an audit probability for those income reports. If an income report gets audited, the enforcer incurs a cost c. In the situation with predation, if a tax evader who has reported x^i is detected to have income y^k, his fine $f(y^k, x^i)$ is net income after taxes minus a grace level $D(y^i, x^i)$. Initially, we assume that the grace level is constant and small, i.e. it satisfies $D < (1-\tau)y^1$.

Citizens maximize expected income and tax enforcers maximize expected net receipts from enforcement. A citizen with income y^k chooses a (mixed) reporting strategy which is a vector $\boldsymbol{\beta}^k = (\beta(y^k, x^i))_{x^i \in Y}$ with $\sum_{x^i \in Y} \beta(y^k, x^i) =$

[7] Our results generalize to the case where there is a minimum income level , $y^0 > 0$, which is exempt under the fine and under the tax scheme.

[8] As strategy choices are constrained to satisfy the the equilibrium conditions, our results do not change if we allow for assignments of competing audition rights.

1.[9] The probability that x^i is announced if strategy vector $\beta(., x^i)$ is used is

$$h^i = \sum_{y^k \in Y} n^k \beta(y^k, x^i)$$

The probability that an agent is of type y^k when she announces x^i is

$$q(y^k|x^i) = \frac{\beta(y^k, x^i)n^k}{h^i}. \tag{1}$$

Tax enforcers choose probabilities $\mathbf{p} = (p(x^i))_{x^i \in Y}$. The (expected) cost of auditing income x^i with a probability $p(x^i)$ is $p(x^i)h^i c$. For a tax enforcer in charge of auditing x^i, the expected net profit from selecting an audition probability $p(x^i)$ is

$$\pi(x^i)(\boldsymbol{\beta}, \mathbf{p}) = h^i p(x^i) K(x^i), \tag{2}$$

where $\boldsymbol{\beta} = (\boldsymbol{\beta}^k)_{0 \leq k \leq N}$ and

$$K(x^i) = \sum_{y^k \in Y} q(y^k|x^i)\, f(y^k, x^i) - c, \tag{3}$$

the marginal gain from auditing a citizen who reports x^i. A citizen's objective function is $E^k(\boldsymbol{\beta}, \mathbf{p}) = y^k - \sum_{x^i \leq y^k} \beta(y^k, x^i)\tau x^i - \sum_{x^i < y^k} \beta(y^k, x^i) \times p(x^i) f(y^k, x^i)$. A strategy configuration is a Nash equilibrium if

$$\pi(x^i)(\boldsymbol{\beta}^*, p(x_i)^*, \mathbf{p}^*_{-i}) \geq \pi(x^i)(\boldsymbol{\beta}^*, p(x_i), \mathbf{p}^*_{-i}) \text{ for all } i$$

and

$$E^k(\boldsymbol{\beta}^*, \mathbf{p}^*) \geq E^k(\boldsymbol{\beta}^k, (\boldsymbol{\beta}^{-k})^*, \mathbf{p}^*) \text{ for all } \boldsymbol{\beta}^k \text{ and for all } k.$$

It is easy to see that for $h^i > 0$ and in the absence of constraints on the detection technology a government agent chooses $p(x^i) = 1$ if $K(x^i) > 0$ and $p(x^i) = 0$ if $K(x^i) < 0$. If $h^i = 0$ or $K(x^i) = 0$ it may select any detection probability. In order to focus on such x^i which are actually reported we need

Definition *A report x^i is in the support of $\boldsymbol{\beta}$, or $x^i \in S(\boldsymbol{\beta})$, if $h^i(\boldsymbol{\beta}) > 0$.*

[9] Citizens with the same income select the same mixed strategy. We are not concerned here with the purification of mixed strategy equilibria.

3 Predatory equilibrium

Given detection probabilities $p(x^i)$ and $p(x^j)$ we denote indifference between two signals x^i and x^j for income group y^k as $(x^i, p(x^i)) \sim_{y^k} (x^j, p(x^j))$. A citizen i is indifferent between reporting x^i and truthfully reporting y^k if

$$p(x^i)D + (1 - p(x^i))(y^k - \tau x^i) = (1 - \tau)y^k. \tag{4}$$

(4) equates expected income with report x^i and the certain net income with the truthful report y^k. It defines for each income the indifference set corresponding to the security level which this agent can ensure in the game by telling the truth. It is convenient to extend the domain of the indifference relation to define indifference contours on (x, p) which are continuous in $(R_0^+)^2$ and each of which contains one indifference set. Indifference contours are concave shaped, decrease in y and intersect the x-axis at $(y^k, 0)$. For $D > 0$ the contour relating to an agent's security level intersects the p-axis at $(0, \overline{p}(y^k, D))$ with $\overline{p}(y^k, D) > \tau$. Security contours of agents with higher income have a smaller p-intersect. Security contours intersect at a signal level ξ with $p(\xi) = \tau$. For $D = 0$, $\xi = 0$. Higher indifference contours are located left below the security contour. For any two indifference contours passing through some (x', p'), $x' > 0$, the one relating to the lower income citizen is steeper. By the following lemma, indifference sets for two individuals intersect at most in one point other than x^0:

Lemma 1 *For x^i: $0 < x^i < y'$, x^j: $x^i < x^j < y'$ and constant D: If at given auditing probabilities $p(x^i)$, $p(x^j)$ a citizen with income y' is indifferent between reporting x^i and x^j all citizens with income $y'' > y'$ strictly prefer reporting x^j over x^i.*

Proof. y' weakly prefers x^j over x^i if $p(x^j)D + (1 - p(x^j))(y' - \tau x^j) \geq p(x^i)D + (1 - p(x^i))(y' - \tau x^i)$. Rearranging yields $(p(x^j) - p(x^i))D \geq (p(x^j) - p(x^i))(y - \tau xi) + (1 - p(x^j))(x^j - x^i)$. For $D < y - \tau x^i$ this implies $p(x^j) < p(x^i)$. The claim is confirmed by substituting $y'' > y'$ in the indifference condition. ■

If for one income group y^{k^*} truth-telling and reporting some x^i are weakly preferred over all other strategies then all other y have strict preferences over reporting x^i or truth-telling. The proof of the following lemma is in the appendix and uses the fact that security contours intersect at $(\xi, p = \tau)$.

Lemma 2 *For $x^i > 0$, $y'' > y'$ and constant D: If at $p(x^i)$, y' is indifferent between truth-telling and reporting x^i then (a) if $p(x^i) < \tau$, y'' strictly prefers reporting x^i; (b) if $p(x^i) = \tau$, y'' is also indifferent and (c) if $p(x^i) > \tau$, y'' strictly prefers truth-telling.*

Let I^k be the set of reports which a citizen with income y^k is prepared to submit, i.e. which maximize her utility.

Lemma 3 *Let D be a constant. Then in equilibrium there can be at most one income group y^{k^*} for which there is a report $x^i > \xi$ and $x^i, y^k \in I^k$.*

Proof. Let there be an income report $x^i \in (0, y^{k^*})$ and an audit probability $p(x^i)$ such that $y^{k^*} \sim_{y^{k^*}} (x^i, p(x^i))$ and $x^i, y^{k^*} \in I^{k^*}$.

a) Assume that there is an income report $x' \in (0, y')$ such that for $y' > y^{k^*}$ we have $y' \sim_{y'} (x', p(x'))$. Then by lemma 2 $(x^i, p(x^i)) \succ_{y'} y'$. Reporting x^i at $p(x^i)$ is feasible and, therefore, $y' \notin I'$.

b) Next, consider $y'' < y^{k^*}$. Say there is $y'' \sim_{y''} (x'', p(x''))$ for some $x'' \in (0, y')$. But because $x'' < y^{k^*}$ the result of part a) equally applies for y'' and $y^{k^*} \notin I^{k^*}$, a contradiction. ■

For $D = 0$ we have $\xi = 0$ such that everyone is indifferent between truth-telling and reporting x^0 if $p(0) = \tau$. For c sufficiently small, we can construct an equilibrium for this game with a pivotal income group y^{k^*}. Citizens with $y < y^{k^*}$ are either honest or report x^0. Citizens with $y > y^{k^*}$ are tax evaders.

Proposition 4 *Suppose that $D \equiv 0$. We focus on equilibria where there is a highest report x^m, such that $x \in S(\boldsymbol{\beta})$ for all $x \leq x^m$ and $x \notin S(\boldsymbol{\beta})$ for $x > x^m$. An equilibrium for this income reporting game is characterized by an threshold income level y^{k^*} and an assignment of indifference sets I^k which satisfies the constraints*

$$
\begin{aligned}
I^N &= \{x^{r^{N-1}}, .., x^{k^*}\} \text{ and } \sum\nolimits_{x^i = x^{r^{N-1}}}^{x^{k^*}} \beta(y^N, x^i) = 1, && (5)\\
I^k &= \{x^{r^{k-1}}, .., x^{r^k}\} \text{ and } \sum\nolimits_{x^i = x^{r^{k-1}}}^{x^{r^k}} \beta(y^k, x^i) = 1\text{: } k^* < k < N, && (6)\\
I^{k^*} &= \{x^0, .., x^{r^*}, y^{k^*}\} \text{ and } \beta(y^{k^*}, y^{k^*}) + \sum\nolimits_{x^i = x^0}^{x^{r^*}} \beta(y^{k^*}, x^i) = 1, && (7)\\
I^j &= \{x^0, y^j\} \text{ and } \beta(y^j, y^j) + \beta(y^j, x^0) = 1 \text{ for } j < k^*, && (8)\\
\beta(y^k, x^i) &\in [0, 1] \text{ for } y^k \neq x^i \text{ and } \beta(y^k, y^k) > 0 \text{ for } y^k \leq y^{k^*}, && (9)\\
K(x^i) &= 0 \text{ for } x^0 < x^i \leq x^{k^*}, && (10)\\
p(x^i) &\geq 0 \text{ for } x^0 \leq x^i \leq x^{k^*} \text{ supports } I^k \text{ for all } k \in N. && (11)
\end{aligned}
$$

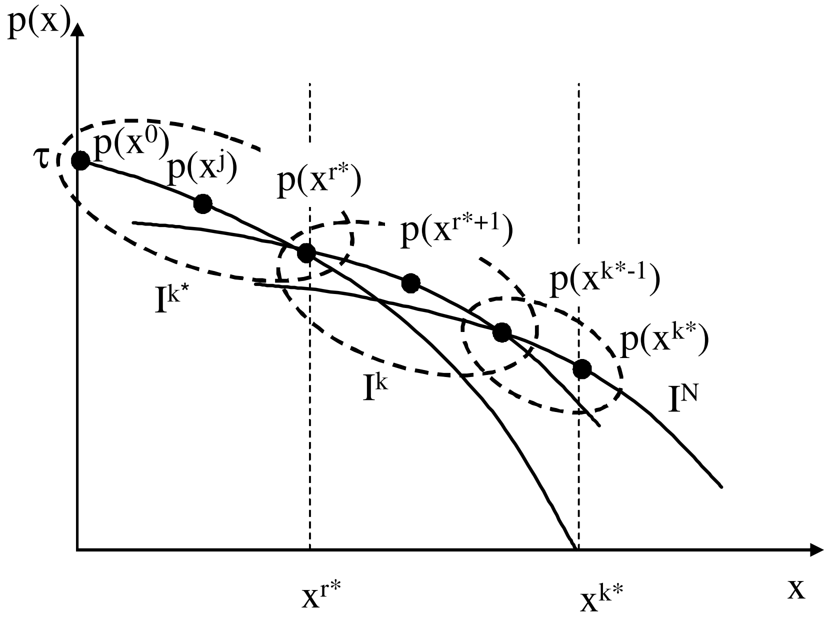

Figure 1: Indifference contours in (x, p)-space and potential reports I^k, supporting an equilibrium $(\boldsymbol{\beta}, \mathbf{p})$ of the income reporting game.

In such an equilibrium all agents with $y > y^{k^*}$ *evade taxes, all agents with* $y^k < y^{k^*}$ *either report honestly or report* x^0*.* $p(x^0) = \tau$ *and* $p(x)$ *decreases in* x *. This equilibrium is unique up to the selection of* y^{k^*} *and* $\beta(y^k, y^k)$ *for* $y^k \leq y^{k^*}$*. The equilibrium exists if* c *satisfies the constraints*

$$c \;<\; \frac{\sum_{y^k > x'}^{y^N} n^k \left(y^k - \tau x'\right))}{\sum_{y^k > x'}^{y^N} n^k} \text{ for all } x' > \min(x^{k^*}, x^{N-1}) \tag{12}$$

$$c \;<\; \frac{\sum_{k \geq 1}^{k^*-1} n^k \beta(y^k, y^k) y^k}{\sum_{k \geq 0}^{k^*-1} n^k}. \tag{13}$$

The proof of this proposition is in the appendix. Figure 1 illustrates the equilibrium conditions: In an equilibrium satisfying proposition 4 the set of potential reports I^k for any two groups of the rich $(y^k > y^{k^*})$ with adjacent income levels overlap in one report. c has to be smaller than average income for the poor $(y^k < y^{k^*})$ by (13) and smaller than average net income for the rich if they report some income x' exceeding x^{k^*}, the income of group k^* by (12). The equilibrium is supported by beliefs which ensure that everyone who submits a report exceeding x^{k^*} is audited. In a Bayes-Nash perfect equilibrium such a belief held by the government would be that the probability of receiving such a report from type y^N is 1. We can construct many equilibria of the type of proposition 4 by perturbing the reporting probabilities $\beta(y^k, y^k)$ of citizens with income below y^{k^*}, ensuring $K(x^0) = 0$. Different choices of $\beta(y^k, y^k)$ may be compatible with different choices of y^{k^*}. The source of this multiplicity is the fact that with $D \equiv 0$ in an equilibrium everyone may submit a report x^0. Below we show that with some arbitrarily small uncertainty about the magnitude of the punishment this source of multiplicity is eliminated. Another source of a possible multiplicity of equilibria is the magnitude of detection cost c. While an equilibrium always exists unless c is prohibitive, there is an intermediate range for c where the equilibrium conditions can be fulfilled by more than one choice of k^*. However, this ambiguity vanishes for sufficiently small c.

Proposition 5 *For sufficiently small c the equilibrium is unique with $k^* = N$ up to a selection of $\beta(y^k, x^0)$, $y^k < y^{k^*}$.*

Proof. See part 3 of the appendix. ■

The following proposition shows that for a grace value $D > 0$, only equilibria are possible where there is $\overline{y} < y^{k^*}$ such that all $y^k < \overline{y}$ report x^0, $\overline{y}$ is indifferent between being honest and reporting x^0 and everyone with $y^k > \overline{y}$ strictly prefers being honest over reporting x^0. For vanishing D we get a refinement of the equilibria compatible with proposition 4.

Proposition 6 *Say an equilibrium exists for $D = 0$. Now let $D : 0 < D < (1-\tau)y^1$. Then in every equilibrium there is $\overline{y}$ such that $\beta(y^k, x^0) = 1$ for all $y^k < \overline{y}$, $\beta(y^k, x^0) \leq 1$ for $y^k = \overline{y}$ and $\beta(y^k, x^0) = 0$ for $y^k > \overline{y}$. Furthermore, these strategies are equilibrium strategies. Letting $D \to 0$, the resulting limit equilibrium is also an equilibrium of proposition 4.*

Proof. See part 4 of the appendix ■

The following proposition is immediate from propositions 5 and 6:

Proposition 7 *For sufficiently small c and $D \to 0$ there is a unique predatory equilibrium.*

4 Is there a role for commitment?

Say King John considers making a promise to relieve some citizens from the most severe punishment. This may take the form of raising D for all his subjects or to announce a shielded income $D(y^i, x^i)$ which depends on true and reported income. The king would do so if under such a regime he expected an increase in tax revenue.

We have already shown that imposing a small punishment relief, $D = \varepsilon \longrightarrow 0$ eliminates equilibria from the tax evasion game. We now verify whether revenue superior equilibria become available once we introduce commitment to a discriminatory punishment relief scheme in the limit equilibrium of proposition 6. We verify whether deviating from such an equilibrium by choosing $D(y^k, x^i) > \varepsilon$ for a single report/income pair (x^d, y^j) is an optimal strategy for the king. What we have to achieve is to make one agent $\overline{y} < y^j < y^{k^*}$ indifferent between truth-telling and saying x^d, $0 < x^d < y^j$. We then adjust randomization probabilities for y^j and y^{k^*} to satisfy the equilibrium conditions. Say in the limit equilibrium we have $(x^d, p(x^d), \varepsilon) \sim_{k^*} (x^{d+1}, p(x^{d+1}), \varepsilon) \sim_{k^*} ... \sim_{k^*} y^{k^*}$. First, observe that we cannot lower the probability of $p(x^d)$ or y^{k^*} would strictly prefer reporting x^d over reporting honestly. Therefore, we adjust $D(y^j, x^d)$, i.e. the income which is left for y^j if she reports x^d and is detected to ensure that $(x^d, p(x^d), D) \sim_{y^j} y^j$. It is easy to show that for this to be the case the condition $D(y^j, x^d) = \frac{\tau}{p(x^d)}(y^j - x^d)$ must hold. For y^{k^*}, we revoke $D(y^{k^*}, x^d) = \varepsilon$, so an agent of income class y^{k^*} does no longer consider submitting income report x^d. Clearly, such a policy avoids introducing additional multiplicity of equilibria. In part 5 of the appendix (lemma 15), we show that if all types have equal probability such a policy is also optimal to replace all x^d income reports from y^{k^*} by reports from y^j, whenever it is optimal to replace some reports. Furthermore, we only consider cases where c is sufficiently small such that $\beta(y^{k^*}, x^d)' = 0$ is compatible with equilibrium[10]. For simplicity, let also $y^{k^*} = y^N$. We can use definitions (1) and (3) to get

$$\beta(y^{k^*}, x^i) = \frac{1}{n^{k^*}} \frac{n^i c}{y^{k^*} - \tau x^i - c} \text{ for } i = d, j \tag{14}$$

in the original equilibrium with $D \longrightarrow 0$. In the new equilibrium

[10] If the condition $K(x^i)$ cannot be met this way, one would want to set $D(y^j, x^d)$ such that all y^j strictly prefer x^d to truthtelling and leave $D(y^{k^*}, x^d) = \varepsilon$.

$$\beta(y^j, x^d)' = \frac{1}{n^j} \frac{n^d c}{y^j - \tau x^d - D(y^j, x^d) - c}, \tag{15}$$

$$\beta(y^{k^*}, x^j)' = \frac{1}{n^{k^*}} \frac{(1 - \beta(y^j, x^d)')n^j c}{y^{k^*} - \tau x^j - c} \tag{16}$$

must hold. (14) ensures that an enforcer is willing to verify any report in the x^i-category at a cost of c when the probability that a report is truthful is $n^i/(\beta n^{k^*} + n^i)$ and the probability that a report submitted by a tax evader with a true income of y^{k^*}, promising the enforcer a fine of $y^{k^*} - \tau x^i$, is $\beta n^{k^*}/(\beta n^{k^*} + n^i)$. In the new equilibrium, y^j replaces y^{k^*} in submitting reports in the x^d-category according to (15). The equilibrium condition (16) for x^j-reports submitted by y^{k^*} displays an indirect effect of increasing $\beta(y^j, x^d)$, as the reduction in the equilibrium probability of honest reports of x^j drives out some of the dishonest reports of x^j by y^{k^*}. Tax paid by y^{k^*} and y^j in the original equilibrium is $\tau[(1 - \beta(y^{k^*}, x^d) - \beta(y^{k^*}, x^j))n^{k^*} y^{k^*} + \beta(y^{k^*}, x^j)n^{k^*} x^j + \beta(y^{k^*}, x^d)n^{k*} x^d + n^j y^j]$ and in the new equilibrium $\tau[(1 - \beta(y^{k^*}, x^j)')n^{k^*} y^{k^*} + \beta(y^{k^*}, x^j)' n^{k^*} x^j + \beta(y^j, x^d)' n^j x^d + (1 - \beta(y^j, x^d)' \tau n^j y^j]$. The change in tax is positive if

$$\begin{aligned} \Delta \; = \; & \beta(y^{k^*}, x^d)n^{k^*} y^{k^*} + [\beta(y^{k^*}, x^j) - \beta(y^{k^*}, x^j)']n^{k^*}(y^{k^*} - y^j) \\ & - \beta(y^j, x^d)' n^j y^j + [\beta(y^j, x^d)' n^j - \beta(y^{k^*}, x^d)n^{k*}]y^d > 0. \end{aligned} \tag{17}$$

It makes sense to restrict attention to revenue superior equilibria with commitment which satisfy an additional stability requirement:

Definition 8 *An equilibrium is deterring if for any two income groups y^j, y^k, $y^k > y^j$ and any feasible report x^i there is no coalitional deviation which increases $\beta(y^j, x^i)$ to some level $\overline{\beta}(y^j, x^i) \leq 1$ such that $K(x^i) < 0$.*

It is easy to see that equilibria of proposition 4 are deterring and that revenue superior equilibria where the construction satisfies conditions (1), (2) and (3) are also deterring. The reason why we introduce this refinement at this point is that punishment relief invites citizens with lower incomes such as y^j to replace citizens with higher income y^{k^*} in submitting a dishonest report

x^i in equilibrium. There could be equilibria which unravel if the invitation is taken up by sufficiently many citizens: if $y^j - \tau x^i - D < c$ no tax enforcer will ever want to investigate x^i even if he knows almost for sure[11] that a citizen who reports x^i is a tax evader with income y^j. It is possible for the revenue superior equilibria to depend on sufficiently few citizens of lower income y^j to take up the invitation (this might also be an optimal policy if conditions (1) and (2) are violated). The condition $K(x^i) = 0$ may be fulfilled if higher income earners y^k submit reports of x^d. But clearly, such equilibria would not be very interesting if they unravel once the unattractive targets of tax enforcers take up the invitation in greater numbers.

The following proposition gives conditions under which a revenue superior equilibrium with partial punishment adjustment exist:

Proposition 9 *If all types have equal probability n^i and for sufficiently small c and τ, there always exist income levels y^{k^*}, y^j and y^d such that $\Delta > 0$. Moreover, the equilibrium reached in the new game with partial commitment is deterring.*

Proof. See appendix. ∎

It should be noted that this result is quite theoretical as with vanishing cost the probability of a true report converges to 1 under any regime. The following example shows that the increase in tax from punishment relief is not unsubstantial:

Example 10 *Let $y^0 = 0$, $y^1 = 1$, $y^2 = 1.4$, $y^3 = 2$. Total income is 4.4 and all types have equal probability. The tax rate $\tau = 0.5$ and detection cost $c = 0.3$. Consider an equilibrium of proposition 4 where $y^{k^*} = 2$ with strategies $\beta(y^1, x^0) = 0.4286$, $\beta(y^2, y^2) = 1$, $\beta(y^3, x^1) = 0.1429$ and $\beta(y^3, x^2) = 0.3$ and detection probabilities $p(x^1) = 0.3333$ and $p(x^2) = 0.2308$. Sheltering assets worth 0.3 from punishment if y^2 is detected when she has reported x^1 raises tax receipts from 1.8243 to 1.8329. The equilibrium with punishment relief is supported by strategies $\beta(y^2, x^1) = 0.5714$ and $\beta(y^3, y^2) = 0.1286$.*

The following table sums up our results for different levels of c:

c	0.0001	0.1	0.2	0.3	0.4	0.5
Tax rev. old	2.199893584	2.0877	1.9628	1.8243	1.6727	1.5125
Tax rev. new	2.199893585	2.0883	1.9659	1.8329	1.6889	1.5125

[11] The cost of auditing most likely consists of the cost of visiting the citizen and the cost of ascertaining the truth.

We observe that the gains from commitment remain positive but vanish as detection cost gets very small. For detection costs of $c = 0.5$ a cornersolution is obtained. For intermediate values, gains in tax receipts can reach 1 percent for c=0.4.

5 Discussion and Conclusion

This paper has shown that a tax reporting game with a predatory government relying on decentralized tax enforcement with maximally feasible fines generically displays multiplicity of equilibria. Introducing a small but general punishment relief significantly reduces the number of equilibria. If detection costs are also small, there exists a unique limit equilibrium with punishment relief. Furthermore, we have shown for a limiting case that granting punishment relief is a way of increasing government revenue. In particular, this is the case if income levels are sufficiently widely spread, types have equal probability and detection costs and tax rates are small. Whilst general existence can be shown under those conditions, gains from commitment are vanishingly small. However, we provide examples where the gains from commitment are substantial.

A few remarks are due on the assumptions we make to derive our results: Whilst it would appear as if assuming away fines as a source of income for the king unduly advances the case for penalty reduction, our results are more general than the particular setting suggests. With linear detection costs, the tax enforcement activity always creates zero profits ex post. Moreover, despite our assumption that decisions are decentralized to each individual tax enforcer, nothing would change if it was the sheriff who acted as a profit maximizer. The equilibrium conditions we derive continue to be relevant if one maximizes aggregate profits. The same results would hold even if the king himself were to enforce his tax laws, provided that he cannot commit himself to impose other than the ex post optimal detection policy.

Needless to say that our results critically hinge on how we define the feasible set on which the maximum penalty is defined. When unbounded penalties induce unbounded negative returns, even the first best solution may be implemented. On the other hand, decision makers are typically willing to accept fatality risks in particular when the probability is sufficiently small. In addition, actually meting out the harshest punishments requires some sort of commitment on the side of the enforcers. Once we accept that pay offs are bounded, it makes sense to identify the feasible set with the positive income and the assets which the government can seize and which otherwise could be taxed.[12] This has the advantage that fines and taxes are defined on the same opportunity set. Moreover, this arrangement satisfies the ulterior fairness test in dividing resources between the citizen and the state of letting Caesar have what belongs to him. Finally, in our motivating story, we assert the death penalty to be infeasible because villagers always have the option to escape into the dark woods of Sherwood forest.

Actually playing the equilibrium of this game, even if it is unique, requires a high degree of coordination on the side of citizens. Full coordination might be difficult to achieve (see Alm/McKee, 2004). In particular, the use of randomized strategies might be seen as counterintuitive. However, if agents have different preferences for tax evasion, it may be possible to support equilibria in purified strategies. Finally, the assumption of risk neutral citizens is of course restrictive. To the extent that risk aversion makes poor citizens relatively more reluctant to evade taxes it works against the strategy configuration in the unique limit equilibrium. However, the set of equilibria described in proposition 4 would still provide some guidance as to which strategy configuration to expect. Our conjecture is that risk aversion would favour an equilibrium where all $y < y^{k^*}$ strictly prefer reporting truthfully.

References

[1] Alm, J., and M. McKee (2004). Tax compliance as a coordination game. *Journal of Economic Behavior and Organization* **54**, 297-312.

[2] Becker, G.S. (1968), Crime and punishment: An economic approach. *Journal of Political Economy* **76**, 169-217.

[3] Boadway, R., and W. Richter (2005). Trading off tax distortion and tax evasion. *Journal of Public Economic Theory* **7**, 361-381.

[4] Borck, R. (2004), Income tax evasion and the penalty structure, *Economics Bulletin* **8**, 1-9.

[12] We ignore issues arising from consumption between the time of reporting income and audition. Making citizens pay penalties exceeding their assets requires future income to be monitored and raises issues of evading payments.

[5] Dhami, S., and A. al-Nowaihi (2006). Hang'em with probability zero: Why does it not work? working paper, University of Leicester.

[6] Dittmann, I. (2006), The optimal use of fines and imprisonment if governments do not maximize welfare, *Journal of Public Economic Theory* **8**, 677-695.

[7] Kolm, S.-C. (1973), A note on optimum tax evasion. *Journal of Public Economics,* **2**, 265-270.

[8] Landsberger, M., D. Monderer and I. Talmor (2000). Feasible net income distributions under income tax evasion: An equilibrium analysis. *Journal of Public Economic Theory* **2**, 135-153.

[9] Polinsky, A.M., and S. Shavell (2000). The Economic Theory of Public Enforcement of Law, *Journal of Economic Literature* **38**, 45-76.

[10] Pyne, D. (2004), Can making it harder to convict criminals ever reduce crime? *European Journal of Law and Economics* **18**: 191-2001.

[11] Reinganum, J., and L. L. Wilde (1986). Equilibrium verification and reporting policies in a model of tax compliance. *International Economic Review* **27**, 739-760.

6 Appendix

6.1 Proof of lemma 2

Let there be an income report $x^i \in (0, y^{k^*})$ and an audit probability $p(x^i)$ such that $y^{k^*} \sim_{y^{k^*}} (x^i, p(x^i))$ and $x^i, y^{k^*} \in I^{k^*}$. Now consider $y' > y^{k^*}$. We had denominated $(\xi, p(\xi) = \tau)$ the intersection of security contours, i.e. for y' and y^{k^*} we have $p(\xi)D + (1-p(\xi))(y - \tau\xi) = (1-\tau)y$. For $D = 0$, we have $p(x^i) < p(\xi)$, $x^i > \xi$. Because the security contour of y' passes right above $(x^i, p(x^i))$, y' strictly prefers $(x^i, p(x^i))$ over truth-telling. For $D > 0$ we find that if $p(x^i) = p(\xi)$ then $\xi = x^i \equiv \widehat{x}$ and x^i is on the security contour of both y' and y^{k^*}. If $p(x^i) < p(\xi)$, then $\xi < x^i$ and y' prefers $(x^i, p(x^i))$ to telling the truth. Finally, if $p(x^i) > p(\xi)$, then $\xi > x^i$ and the security contour passes left below x^i, i.e. y' prefers truth-telling.

Next, consider $y'' < y^{k^*}$. For $D = 0$, $p(x^i) < 0$, $\xi < x^i$, the security contour of y'' passes right below $(x^i, p(x^i))$ and y'' strictly prefers truth-telling. The same is true for $D > 0$, $p(x^i) < p(\xi)$. For $D > 0$, $p(x^i) \geq 0$, truth-telling is dominated for y''. Collecting arguments, if $p(x^i) < p(\xi)$ all $y > y^{k^*}$ strictly prefer lying over truth-telling and all $y < y^{k^*}$ strictly prefer truth-telling over lying and the relationships are reversed for $p(x^i) > p(\xi)$.

6.2 Proof of proposition 4

Before we establish the proposition we need to prove two claims:

Claim 11 *$p(x^0) > \tau$ is not an equilibrium*

Proof. Suppose $p(x^0) > \tau$ were true. In that case everybody rather reports truthfully than reporting x^0. But then $h^0 = n^0 > 0$ and $K(x^0) < 0$ so the tax enforcer picks $p(x^0) = 0$, a contradiction.□

Claim 12 *Let $p(x') > 0$ and $x', x'' \in S(\boldsymbol{\beta})$. Then, in equilibrium it must be that $p(x'') < p(x')$ if $x'' > x'$ for all $x' \geq 0$.*

Proof. Suppose that not. Then for all $y^k > x''$ it is the case that $x' \succ_{y^k} x''$. But then $K(x'') < 0$ and because $x'' \in S(\boldsymbol{\beta})$ the tax enforcer sets $p(x'') = 0$, a contradiction.□

From claim 12 follows immediately that $K(x^i) = 0$ for all $x^i \in S(\boldsymbol{\beta})$, $x^i \leq x^m$. Suppose that not. Then $p(x^i)$ is either 0 or 1. The latter case is trivial given claim 11 and 12, so focus on $p(x^i) = 0$. In that case, no $x' \geq x^i$ is audited and it must be that $x^i > x^m$, contradicting that $x^i \in S(\boldsymbol{\beta})$.

We now construct equilibria satisfying $K(x^i) = 0$, $\forall x^i \in S(\boldsymbol{\beta})$. All $x^i \in S(\boldsymbol{\beta})$ must be named in some false reports. By lemma 3 there can be only one income y^{k^*} such that $y^{k^*} \in I^{k^*}$ and $x^i \in I^{k^*}$ for some $x^i \neq x^0$. Therefore, $y^k < y^{k^*}$ prefer reporting truthfully over reporting $x > x^0$ and $y^k > y^{k^*}$ prefer submitting a false report over reporting y^k. By lemma 2, I^k and I^j, $k \neq j$ intersect at most in one point and in general $I^{y^k} \cap I^{y^j} \neq \emptyset$ if we want to fulfill $\sum_{x^i \in I^k} \beta(y^k, x^i) = 1$ for a tax evader (i.e. $y^k > y^{k^*}$). Note that our definition of equilibrium includes the case where $I^k \cap I^j = \emptyset$ as we can always set $\beta(y, x^r) = 0$ for one agent. Finally, by (13) we can fulfill $K(x^0) = 0$ only using agents with $y < y^{k^*}$.

Claim 13 *$x^m = \min(x^{k^*}, x^{N-1})$, where x^{k^*} is the truthful report of y^{k^*}, and $x^{k^*} \in I^N$.*

Proof. Note that $x^m \in S(\boldsymbol{\beta})$. If $x^m > x^{k^*}$, $\beta(y^m, y^m) = 0$ by lemma 1. Suppose that $x^m = x^{N-1} > x^{k^*}$. Then $K(x^{N-1}) = y^N - \tau x^{N-1} - c > 0$ for all $\beta(y^N, x^{N-1}) > 0$. By (12), $K(x^{N-1}) > 0$, so $p(x^{N-1}) = 1$. Proceeding to $x^m = x^{N-2}$, .., x^{k^*+1}, we find that for any $x^m > x^{k^*}$ having $c < \frac{\sum_{y^k > x^m}^{y^N} n^k \left(y^k - \tau x^m\right)}{\sum_{y^k > x^m}^{y^N} n^k}$ fulfilled subsequently for all $m = N-2, N-3, ...$ is sufficient to ensure that $K(x^m) > 0$. This is condition (12). Because x^N is never audited, $x^m > x^{k^*}$ is impossible if $x^{k^*} = x^N$ or $x^{k^*} = x^{N-1}$. Suppose the sign in (12) is reversed. Then $K(x^m) < 0$ and $p(x^m) = 0$.

If $x^{k^*} \notin I^N$ then no one submits a false report x^{k^*}, so $K(x^{k^*}) < 0$. Finally, we have to check that it is actually optimal for y^N not to submit a report $x' > x^{k^*}$: As by claim 11, $p(x^0) \leq \tau$ such a report is clearly dominated.□

By the following claim we can actually construct profiles $p(x^i)$ to support I^k:

Claim 14 *The constraint $p(x) \geq 0$ is never binding.*

Proof. Let $x^{k^*-1} \in I^{k^*} \cap I^{k^*+1}$. Because the contour corresponding to I^{k^*+1} cuts from below I^{k^*} at x^{k^*-1} and I^{k^*} contains the point $(p = 0, x^{k^*})$, it must be that $p(x^{k^*}) > 0$ to have $x^{k^*} \in I^{k^*+1}$. The same is true if x^{r^*+1} is reached via a sequence of indifference conditions for $x^{r^*+1} \in I^{k^*+1} \cap I^{k^*+2}, .., x^{k^*-1} \in I^{N-1} \cap I^N$ where the contour corresponding to the set with the higher index cuts the other from below (see the graph).□

6.2.1 Existence and uniqueness

Choose $\beta'(y, x^0)$ and $\beta'(y, y)$, $y < y^{k^*}$ such that $K(x^0) = 0$. By (13) such a profile exists. Here we set $\beta(y^{k^*}, x^0) = 0$. (5)-(7) give the following system $\boldsymbol{\gamma} = \mathbf{B}\boldsymbol{\beta}^*$ with

$$\boldsymbol{\gamma} = \begin{pmatrix} 1 \\ 1 \\ \beta'(y^k, y^k) \\ 1 \\ \beta'(y^{r^*+1}, y^{r^*+1}) \\ 1 \\ \beta'(y^{r^*}, y^{r^*}) \end{pmatrix}, \boldsymbol{\beta}^* = \begin{pmatrix} \beta(y^N, x^{k^*}) \\ \beta(y^N, x^k) \\ \beta(y^{N-1}, x^k) \\ \beta(y^{N-1}, x^{r^*+1}) \\ \beta(y^{k^*+1}, x^{r^*+1}) \\ \beta(y^{k^*+1}, x^{r^*}) \\ \beta(y^{k^*}, x^{r^*}) \\ \beta(y^{k^*}, x^j) \end{pmatrix},$$

$$
\mathbf{B} = \begin{pmatrix}
a^{N,k^*} & & & & & & \cdots & 0 \\
1 & 1 & & & & & & \vdots \\
 & a^{N,k} & a^{N-1,k} & & & & & \\
 & & 1 & 1 & & & & \\
 & & & a^{N-1,r^*+1} & a^{k^*+1,r^*+1} & & & \\
 & & & & 1 & 1 & & \\
\vdots & & & & & a^{k^*+1,r^*} & a^{k^*,r^*} & \\
0 & \cdots & & & & & & a^{k^*,j}
\end{pmatrix}
$$

where $a^{k,i} = n^k(y^k - \tau x^i - c)$ and we have used $\beta(y^{k^*}, y^{k^*}) = 1 - \sum \beta(y^{k^*}, x^j)$ in the first row of $\mathbf{B}$ to replace $a^{N,k^*} = \beta(y^{k^*}, y^{k^*})$. For concreteness and in order to save space we have imposed $N = k^* + 3$, taken $\#I^k = 2$ for $y^k > y^{k^*}$, set $\beta(y^{k^*}, x^0) = 0$, and used a typical element $a^{k^*,j}$ for $0 \leq j < r^*$.[13]

$\mathbf{B}$ has full rank. So given k^* and a selection of $\beta(y, y)$ for $y < y^{k^*}$ to satisfy $K(x^0) = 0$ the solution of $\boldsymbol{\beta}^* = \mathbf{B}^{-1}\boldsymbol{\gamma}$ uniquely determines $\boldsymbol{\beta}^*$. Furthermore, given that the $a^{k,i}$'s are all positive and all $\beta(y^i, y^i) > 0$, $y^i < y^{k^*}$, looking at a typical expression, $\frac{n^k\beta^*(y^k,x^i)(y^k-\tau x^i)+n^{k+1}\beta^*(y^{k+1},x^i)(y^{k+1}-\tau x^i)}{n^i\beta'(y^i,y^i)+n^k\beta^*(y^k,x^i)+n^{k+1}\beta^*(y^{k+1},x^i)} = c$ with $c < y^k - \tau x^i$, $n^i \geq 1$, it is easy to see that we can solve the system such that all coefficients in β^* are positive and smaller than 1.

An equilibrium exists for $c \to 0$ with $k^* = N$. Now let $\beta(y^k, x^0) = 0$ for all $0 < k < k^*$ and define $c^{\max}$ the maximum c for which that there is an equilibrium for $k^* = 1$ under this constraint on $\boldsymbol{\beta}$.[14] We have to show that there is an equilibrium for all c', $0 < c' < c^{\max}$. For convenience, we write β_i^k for $\beta(y^k, x^i)$ and y_i^k for $y^k - \tau x^i$. We show that an equilibrium for $c < c^{\max}$ always exists but that in general this equilibrium is not unique. The equilibrium exists if for increasing threshold values, $k^* = J$, $k^{*\prime} = J + 1$, $c^{\max}(J+1) \geq c^{\min}(J)$ and, consequently, there is no c' with $c^{\max}(J+1) < c' < c^{\min}(J)$. The equilibrium is not unique, if the inequality is strict, because there is c'' with $c^{\max}(J+1) > c'' > c^{\min}(J)$ for which an equilibrium with $k^* = J$ or $k^{*\prime} = J + 1$ can be constructed.

Let $k^* = 1$. Choose $I^1 = \{x^0, x^1\}$, $I^N = \{x^0, x^1\}$ and $I^k = \{x^0\}$ for

[13]The extension is straightforward, notably for each added x^i, add one line and one column and for each added $y > y^{k^*}$ add two lines and two columns.

[14]Such defined $c^{\max} \leq \bar{c}$, the greatest c which satisfies (12) and (13).

$k = 2, ..., N-1$, thereby satisfying (5)-(8). At x^0:

$$\frac{\sum_{i=2}^{N-1} n^i y^i + \beta_0^1 n^1 y_0^1 + \beta_0^N n^N y_0^N}{n^0 + \sum_{i=1}^{N-1} n^i + \beta_0^1 + \beta_0^N n^N} = c^0(1)$$

and at x^1:

$$\frac{\beta_1^N n^N y_1^N}{(1-\beta_0^1)n^1 + \beta_1^N n^N} = c^1(1).$$

Determine $\boldsymbol{\beta}$ such that $c^{\min}(k^* = 1) = \min(\max(c^0(1), c^1(1)))$. Clearly, it must be that $\beta_0^1 = 0$. Now let $k^* = 2$. We have $x^1, x^2 \in I^N$ and $x^0, x^1 \in I^{N-1}$. At x^0:

$$\frac{\sum_{i=3}^{N-2} n^i y^i + \beta_0^2 n^2 y_0^2 + \beta_0^{N-1} n^{N-1} y_0^{N-1}}{n^0 + \sum_{i=3}^{N-2} n^i + \beta_0^2 + \beta_0^{N-1} n^{N-1}} = c^0(2).$$

At x^1:

$$\frac{\beta_1^{N-1} n^{N-1} y_1^{N-1} + \beta_1^N n^N y_1^N}{n^1 + \beta_1^{N-1} n^{N-1} + \beta_1^N n^N} = c^1(2)$$

and at x^2:

$$\frac{\beta_2^N n^N y_2^N}{(1-\beta_0^2)n^2 + \beta_2^N n^N} = c^2(2).$$

We can determine $\boldsymbol{\beta}'$ such that $c^i(2) = c^{\min}(1)$, $i = 0, 1, 2$. For this choose $\beta_0^1 = 0$ and $\beta_0^{N-1} = 1$. Because $c^{\max}(2) \geq c^i(2)$ it must be that $c^{\max}(2) \geq c^{\min}(1)$. We can also see that the solution is in general non unique: $c^{\max}(k^* = 2)$ is in general obtained for $\beta_0^{N-1} \neq 1$: Suppose that $\beta_0^{N-1} = 1$ and $n^N y^N \to 0$. Then $\beta_1^{N-1} = 0$ and $c^1(2) < c^0(2)$, contradicting that c is maximal. Because the expression for $c^0(1)$ implies that $\beta_0^{N-1} = 1$, we have $c^{\max}(k^* = 2) > c^{\min}(k^* = 1)$ for $\beta_0^{N-1} \neq 1$.

For the general case we determine $c^{\min}(J)$ for $k^* = J$ and construct $\boldsymbol{\beta}$ for $J+1$ which is feasible and supports $c = c^{\min}(J)$, so $c^{\max}(J+1) \geq c = c^{\min}(J)$. Say that for $k^* = J$ we have found $\boldsymbol{\beta}$ supporting $c^{\min}(J)$ with $I^J = \{x^0, .., x^{r^1}, y^J\}$, $I^k = \{x^{r^{k-1}}, ..., x^{r^k}\}$ for $N \geq k > J$ with $x^{r^N} = x^J$. For concreteness, let $I^J = \{x^0, y^J\}$ and $I^{J+1} = \{x^0, x^1\}$ so we obtain typical expressions such as, at x^0:

$$\frac{\beta_0^J n^J y_0^J + \beta_0^{J+1} n^{J+1} y_0^{J+1}}{n^0 + \beta_0^J n^J + \beta_0^{J+1} n^{J+1}} = c^0(J),$$

at x^1:

$$\frac{\beta_1^{J+1} n^{J+1} y_1^{J+1} + \beta_1^{J+2} n^{J+2} y_1^{J+2}}{n^1 + \beta_1^{J+1} n^{J+1} + \beta_1^{J+2} n^{J+2}} = c^1(J),$$

at x^J:

$$\frac{\beta_J^N n^N y_J^N}{\beta_J^J n^J + \beta_J^N n^N} = c^J(J).$$

In $c^{\min}(J)$ we have $\beta_0^J = 0$, $\beta_J^J = 1$. Now let $k^{*\prime} = J + 1$ with corresponding $I^{J\prime} = \{y^J\}$, $I^{J+1\prime} = \{x^0, x^1, y^{J+1}\}$, $I^{k\prime} = I^k$ for $k < N$ and $I^N = \{x^{J-1}, x^J, x^{J+1}\}$. At x^0:

$$\frac{\beta_0^{J+1} n^{J+1} y_0^{J+1}}{n^0 + \beta_0^J n^J + \beta_0^{J+1} n^{J+1}} = c^0(J+1),$$

at x^1:

$$\frac{\beta_1^{J+1} n^{J+1} y_1^{J+1} + \beta_1^{J+2} n^{J+2} y_1^{J+2}}{n^1 + \beta_1^{J+1} n^{J+1} + \beta_1^{J+2} n^{J+2}} = c^1(J+1),$$

at x^J:

$$\frac{\beta_J^N n^N y_J^N}{n^J + \beta_J^N n^N} = c^J(J+1).$$

at x^{J+1}:

$$\frac{\beta_{J+1}^N n^N y_J^N}{\beta_{J+1}^{J+1} n^{J+1} + \beta_{J+1}^N n^N} = c^J(J+1).$$

Letting $\beta_{J+1}^{J+1} \to 0$ and $\beta_{J+1}^N \to 0$ we can fulfill the condition for x^{J+1} and for all other k $\beta_i^{k\prime} \to \beta_i^k$ so we have $c^i(J+1) \to c^{\min}(J)$. Because $\boldsymbol{\beta}$ was selected to minimize c it must be that $c^{\max}(J+1) > c^i \to c^{\min}(J)$.

For the last step, we consider $k^* = N-1$. The minimum cost is sustained with $I^{N-1} = \{x^0, y^{N-1}\}$ and $I^N = \{x^i\}_{i=0}^{N-1}$. At x^0:

$$\frac{\beta_0^{N-1} n^{N-1} y_0^{N-1} + \beta_0^N n^N y_0^N}{n^0 + \beta_0^{N-1} n^{N-1} + \beta_0^N n^N} = c^0(N-1), \tag{18}$$

at x^i

$$\frac{\beta_i^N n^N y_i^N}{n^i + \beta_i^{N-1} n^{N-1}} = c^i(N-1), i = 1, .., N-2, \tag{19}$$

at x^{N-1}:

$$\frac{\beta_{N-1}^N n^N y_{N-1}^N}{n^{N-1}\beta_{N-1}^{N-1} + \beta_{N-1}^N n^N} = c^{N-1}(N-1). \tag{20}$$

The minimum is attained by setting $\beta_0^{N-1} = 0$ and choosing all β_i^N, $i \neq N$ such that all $K(x^i) = 0$. Switching to $k^* = N$, we have $I^N = \{x^i\}_0^N$. At x^i:

$$\frac{\beta_i^N n^N y_i^N}{n^i + \beta_i^N n^N} = c^i(N), i = 0, .., N-1. \tag{21}$$

β_0^{N-1} is also feasible for $k^* = N$ and we have $c^{\max}(N) \geq c^{\min}(N-1)$.

6.2.2 Show that $p(x^0) = \tau$.

Finally, we show that if the equilibrium of the reporting game satisfies (5)-(13), then $p(x^0) < \tau$ is not compatible with an equilibrium of this game: Suppose $p(x^0) < \tau$ were true. In such an equilibrium, everybody has at least one strategy which is strictly better than truth telling, so everybody evades taxes. If we had $p(x^0) = 0$ everybody would report x^0, so this cannot be an equilibrium. In order to construct an equilibrium with $p(x^0) \in (0, \tau)$ with we need $K(x) = 0$ for all $x \in \{x^0, .., x^m\}$. In an equilibrium satisfying (9) we have $\beta(y^k, y^k) > 0$ for $y^k \leq y^{k^*}$ and, therefore, $\sum_{x^i \leq x^{k^*}, x^i \in S(\boldsymbol{\beta})} \sum_{y^k > x^i} q(y^k|x^i) < 1$. Now consider the equilibrium with $p(x^0) < \tau$. Reports $x^i > x^m$ cannot occur in equilibrium (see the proof of claim 8 in the appendix) so in an quilibrium where everybody evades, $\sum_{x^i \leq x^m, x^i \in S(\boldsymbol{\beta})} \sum_{y^k > x^i} q(y^k|x^i) = 1$. Therefore, $x^m > x^{k^*}$. But this cannot be an equilibrium either under condition (12) because $x^m \in S(\boldsymbol{\beta})$ and, therefore, $p(x^m) = 1$ (see proof of claim 8).

6.3 Proof of proposition 5

Using conditions (18)-(21) in part 2 of the appendix we now show that for c sufficiently small the equilibrium is unique. For $c \to 0$ there is an equilibrium with $k^* = N$ where N's strategy satisfies $\sum_{i=0}^{i=N-1} \beta_i^N \to 0$. Show that in general $c^{\min}(k^* = N-1) > \varepsilon$, $\varepsilon > 0$. Recall that the population is finite and all $n^k \geq 1$ and X is finite as well. From the conditions for $c^i(N-1)$ letting $\beta_0^{N-1} = 0$ we get $c^{\min} = \min(\frac{\beta_0^N}{n^0+\beta_0^N n^N}, ..., \frac{\beta_{N-1}^N n^N y^N}{n^{N-1}+\beta_{N-1}^N n^N})$ which is bounded from zero because $\sum_{i=0}^{i=N-1} \beta_i^N = 1$.

6.4 Proof of proposition 6

For $D > 0$ we have $\overline{p}(y^k, D) > \tau$. Say there is $(x^0, \overline{p}(y^k, D)) \sim_{y'} y'$. Then by lemma 2 all $y'' > y'$ strictly prefer truth-telling to reporting x^0.

Choose the smallest $\overline{y}$ such that $K(x^0) = 0$ with $\beta(\overline{y}, x^0) \leq 1$ and $\beta(y^k, x^0) = 1$ for all $y^k < \overline{y}$ and select $p(x^0)$ to satisfy $p(x^0)D + (1-p(x^0)) = (1-\tau)\overline{y}$. By lemma 2, all $y' > \overline{y}$ prefer truth-telling to reporting x^0 and all $y'' < \overline{y}$ prefer reporting x^0 to truth-telling. This rules out any equilibrium with $\beta(y^k, x^0)' \neq \beta(y^k, x^0)$. Because in an equilibrium of proposition 4 c satisfies (12) and (13) there must $\boldsymbol{\beta}$ and $\overline{y} < y^{k^*}$ with $\beta(y^k, x^0) \leq 1$, $y^k \leq \overline{y}$ such that $K(x^i) = 0$ for all $x^i \in S(\beta)$.

Finally, in order to show that the limit equilibrium is also an equilibrium of proposition 4, recall that this equilibrium satisfies $p(x^0) = \tau$. For $D \to 0$, $\overline{p}(y^1, D) \to \tau = p(x^0)$. The strategy profile $\boldsymbol{\beta}$ for the case $D > 0$ implies that $x^0 \in I^k$ for some $y^k \leq \overline{y}$ and $x^i \in I^{k^*}$ such that $K(x^i) = 0$ for all $x^i \in S(\beta)$. But then $\boldsymbol{\beta}$ satisfies the conditions in proposition 4.

6.5 Proof of proposition 9

In the following we write β_i^k for $\beta(y^k, x^i)$.

Lemma 15 *If all types have equal probability, whenever $\Delta > 0$ for some $\beta^j(y^j, x^d) > 0$, it is optimal to select $\beta^j(y^j, x^d)$ such as to fulfill $K'(x^d) = 0$.*

Proof. Totally differentiating the condition for $K'(x^d) = 0$ we obtain $\frac{d\beta^{k^*}(y^{k^*}, x^d)}{d\beta^j(y^j, x^d)} = -\frac{n^j[N(y^j - \tau x^d - D) - 1]}{n^{k^*}[N(y^{k^*} - \tau x^d) - 1]}$ with $y_d^j = y^{k^*} - \tau x^d - D$ and $N = \beta(y^{k^*}, x^d)n^{k^*} + \beta(y^j, x^d)n^j + const$. Because $y^{k^*} > y^j$, it follows that $y^{k^*} - \tau x^d - D > y^j - \tau x^d$. If group sizes are equal, N increases as we substitute

$\beta(y^j, x^d)$ for $\beta(y^{k^*}, x^d)$ and this reinforces the marginal effect on $\beta(y^{k^*}, x^d)$. The effect on revenue is $\frac{dT}{d\beta^j_d} = \tau[\frac{d\beta^k_d}{d\beta^j_d}(y^k - y^d) + (y^d - y^j) - \frac{d\beta^k_j}{d\beta^j_j}\frac{d\beta^j_j}{d\beta^j_d}(y^k - y^j)$. Here $\frac{d\beta^j_j}{d\beta^j_d} = -1$ and $\frac{d\beta^k_j}{d\beta^j_j}$ is constant such that the claim follows. ■

As the lemma shows, when the government wants to see $\beta(y^j, x^d)$ increased at all, it is optimal to set $D(y^{k^*}, x^d) = 0$. In that case, citizens want to satisfy the equilibrium condition at x^d by adjusting $\beta(y^j, x^d)$. By construction, such an equilibrium, if it exists, is deterring. It remains to show that there generally exist income levels such that Δ is positive.

Combining (14) and (16) we find that $\beta^{k^*}_j - \beta^{k^*\prime}_j = \beta^{k^*}_j \beta^{j\prime}_d$. Recalling the definition $x^j = y^j$, we can re-write (17) to give $\Delta/\tau = [\beta^{k^*}_d + \beta^{k^*}_j \beta^{j\prime}_d] n^{k^*} y^{k^*} - (\beta^{k^*}_j \beta^{j\prime}_d n^k + \beta^{j\prime}_d n^j) y^j + (\beta^{j\prime}_d n^j - \beta^{k^*}_d n^{k^*}) x^d$. Using the indifference condition (4) for y^{k^*} and y^j at x^j and x^d in the expressions (14)-(16) we get

$$\Delta = \frac{n^d c}{\frac{1-\tau}{1-p(x^d)} y^{k^*} - c}(y^{k^*} - y^d) + \frac{c}{\frac{1-\tau}{1-p(x^j)} y^{k^*} - c} \frac{n^d c}{\Xi}(y^{k^*} - y^j) - \frac{n^d c}{\Xi}(y^j - y^d)$$

with $\Xi = \frac{1-\tau}{1-p(x^d)} y^j - \frac{D}{1-p(x^d)} - c$. Recall that $(y^{k^*} > y^j > y^d)$. Deviding by $(y^{k^*} - y^d)$ and defining $\alpha = \frac{(y^j - y^d)}{(y^{k^*} - y^d)}$ we get

$$\Delta > 0 \Longleftrightarrow \frac{n^d c}{\frac{1-\tau}{1-p(x^d)} y^{k^*} - c} + \frac{c}{\frac{1-\tau}{1-p(x^j)} y^{k^*} - c} \frac{n^d c}{\Xi}(1 - \alpha) > \frac{n^d c}{\Xi}\alpha$$

Further, we write

$$\frac{\frac{1-\tau}{1-p(x^d)} y^j - \frac{D}{1-p(x^d)} - c}{\frac{1-\tau}{1-p(x^d)} y^{k^*} - c} + \frac{c}{\frac{1-\tau}{1-p(x^j)} y^{k^*} - c}(1 - \alpha) > \alpha$$

In order to prove the claim we let $c \longrightarrow 0$. After some straightforward manipulation the expression converts into

$$\frac{y^j - \frac{D}{1-\tau}}{y^j - y^d} > \frac{y^{k^*}}{y^{k^*} - y^d}$$

We know $D = \frac{1-\tau}{p(x^d)}(y^j - y^d)$. From the indifference condition of y^{k^*} in the original equilibrium we get $p(x^d) = \frac{\tau(y^{k^*} - y^d)}{y^{k^*} - \tau y^d}$. Inserting the expression for D yields after some transformations

$$\frac{y^j}{y^j - y^d} > \frac{y^{k^*}}{y^{k^*} - y^d} + \frac{1}{1-\tau} \frac{y^{k^*} - y^d}{y^{k^*} - \tau y^d}$$

Now, letting $\tau \longrightarrow 0$ we find the condition

$$\frac{y^j}{y^j - y^d} > \frac{y^{k^*}}{y^{k^*} - y^d} + \frac{y^{k^*} - y^d}{y^{k^*}}$$

If y^{k^*} gets large relatively to y^d, the right hand side converges to 2. So we just have to choose y^d and y^j to ensure that the left hand side gets sufficiently larger than 2, for example $y^j = 5$ and $y^d = 4$.

Maple worksheet for example 10, only for the purpose of verification by the referees

```
> restart;
> y1:=1;
```

$$y1 := 1$$

```
> y2:=1.4;
```

$$y2 := 1.4$$

```
> y3:=2;
```

$$y3 := 2$$

```
> n0:=1;
```

$$n0 := 1$$

```
> n1:=1;
```

$$n1 := 1$$

```
> n2:=1;
```

$$n2 := 1$$

```
> n3:=1;
```

$$n3 := 1$$

```
> c:=0.3;
```

$$c := .3$$

```
> t:=0.5;
```

$$t := .5$$

```
> px0:=t;
```

$$px0 := .5$$

```
> px1:=((y3-t*y1)-((1-t)*y3))/(y3-t*y1);
```

$$px1 := .3333333333$$

```
> px2:=((y3-t*y2)-((1-t)*y3))/(y3-t*y2);
```

$$px2 := .2307692308$$

```
> test:=(t/px1)*(y2-y1)-c;
```

$$test := .3000000000$$

```
> test:=y1/y2-(2*px1-t+((t-px1)/(1-px1)))/(2*px1*t);
```

$$test := -.5357142857$$

```
> b10:=(n0*c)/(n1*(y1-c));
```

$$b10 := .4285714286$$

```
> b10:=b10;
```

$$b10 := .4285714286$$

```
> F31:=y3-t*y1;
```

$$F31 := 1.5$$

```
> b31:=(n1*c*(1-b10))/(n3*(F31-c));
```

$$b31 := .1428571428$$

```
> F32:=y3-t*y2;
```

$$F32 := 1.30$$

```
> b32:=(n2*c*1)/(n3*(F32-c));
```

$$b32 := .3000000000$$

```
> Tax1:=t*((1-b10)*n1*y1+y2*n2+(1-b31-b32)*y3*n3+b31*y1*n3+b32*y2*n3
  );
```

$$Tax1 := 1.824285714$$

```
> d:=y1*(t*(1-px1))/px1-y2*(t-px1)/(px1);
>
```

$$d := .2999999997$$

```
> F21D:=y2-t*y1-d;
```

$$F21D := .6000000003$$

```
> b21:=(n1*c*(1-b10))/(n2*(F21D-c));
```

$$b21 := .5714285708$$

```
> B21:=min(b21,1);
```

$$B21 := .5714285708$$

```
> b32:=(n2*c*(1-B21))/(n3*(F32-c));
```

$$b32 := .1285714288$$

```
> Tax2:=t*((1-b10)*y1*n1+(1-b21)*y2*n2+(1-b32)*y3*n3+b21*y1*n2+b32*y
  2*n3);
```

$$Tax2 := 1.832857143$$

```
> pro:=(Tax2-Tax1)/Tax1;
```

$$pro := .004698512373$$

```
>
>
>
```