Government Formation, Budget Negotiations and Re-election Uncertainty: The Cases of Minority and Majority Coalition Governments.

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ABSTRACT

This paper presents an analytical approach, which connects the form of a government and the level of expenditures, which it runs. It explains the findings on spending patterns of minority and majority coalition governments reported by the empirical literature. We present a two period model in which a government is formed in the first period and budget negotiations in the cabinet and the parliament take place in the beginning of the second period. Parties derive utility from expenditures and office rents. Minority governments emerge in the absence of a government premium. We show that in the first period, minority government only form if there is no office rent and have the same average expenditures as majority coalition governments. In the second period, the head of government
has an incentive to renege on his government. Minority governments which form in the second period where they are predicted to have higher expenditures. The demand of a party in the parliament reflects her re-election prospects. In the absence of political risk, majority coalition and minority governments are predicted not to run different expenditure policies. With a rise in re-election uncertainty, a pre-existing coalition government faces risk of termination, in which case the probability that it is followed by a minority government with higher expenditures increases.
I. Introduction

There is a vast but rather inconclusive empirical literature which relates political arrangements to fiscal policies. Indicators of fragmentation tend to have a positive effect at least on public debt growth (Kontopolous and Perotti, 1997 and De Haan, Sturm and Beekhuis, 1999). The frequency of government changes affects average fiscal policies in the same direction, which suggests that political instability is an issue (De Haan and Sturm, 1994, De Haan, Sturm and Beekhuis 1999).

A weak bureaucracy and institutional arrangements, which leave the head of government in a weak position within the budget process are found to raise expenditures (Von Hagen, 1990, Hahm, Kamlet and Mowery, 1996). Often, but maybe counterfactually, minority governments are associated with a weak government. However, the empirical literature does generally not find systematic differences between spending patterns of minority and majority or majority coalition governments. Earlier contributions had established such a relationship, indicating that minority governments run higher deficits (Roubini and Sachs, 1989a, Corsetti and Roubini, 1993 and Edin and Ohlsson, 1991) and spend more (Roubini and Sachs, 1989b). These earlier results were convincingly rejected by De Haan and Sturm (1997). With an appropriate measure, these effects are not significant. However, we find some cases where a minority government emerged in a situation of political crisis, after the termination of a preexisting majority coalition government. These cases, including the Moro minority government in Italy after 1974, the Swedish Falling minority government after 1981 and the early Danish Jorgenson government after 1981, coincided with exceptionally high government expenditures with a time lag of about one year after gaining power.
On the other hand, minority governments do not need to emerge in a situation of crisis, but may rather be a feature of a stable political environment (see Strom, 1990). This is endorsed by the experience of the Danish Schluter minority government, which was strong enough to call for early elections where it would find support in seeing through its budget reform policy, even without gaining a formal majority. This suggests that while minority governments which emerge in a situation of political crisis do have relatively high expenditures, minority governments, which emerge in a stable situation may even have relatively low expenditures.

In the formal literature, which discusses the choice of the form of government in a bargaining framework, the emergence of a minority government often results from a strong position of the formateur. In Laver and Shepsle (1996) a party is in a position to form a minority government if this party is strong. Strength derives from the preferences of the other parties in relation to the policy the successful minority government is expected to implement. In Baron (1998), a minority government only emerges, if the party who is offered to support a minority government looses income by rejecting the proposal. A different approach is taken by Diermeier and Merlo (1999). They consider a two period model in which the size of the cake depends on the composition of the government. If the formateur dominates the negotiations within the government (i.e. if he is favored by the status quo), he forms a supermajority government. Otherwise, he forms a minority government and makes a side payment to the party whose vote is the cheapest. None of these models, however, lends itself to the analysis of government expenditures in a straightforward way. The approach to government expenditures on which this paper draws is based on the legislative bargaining

In this paper I present a coherent analytical framework, which relates the expected payoffs from entering budget negotiations from outside or inside the government to the options, which are available at the government formation stage. Therefore, issues of the budget policy expected and the form of government, i.e. majority coalition or minority government, can be addressed simultaneously. For this, I develop a multi stage model, where the budgeting game is preceded by a government formation stage. A budget bill has to be adopted by the cabinet (of a minority or majority coalition government) and, subsequently, to be accepted by the parliament. The parliament, upon rejection, may amend the bill or call for new elections.

The demand for budgetary expenditures is driven by some straightforward considerations. First, a higher demand by a party supporting the government raises the level of expenditures. This is satisfied, if one player - i.e. the head of government - dominates the budget procedure as a residual claimant and budget negotiations are modelled as a trade off between spending and taxation\(^1\). Consequently, via raising the demand of the supporting party, expenditures rise with an improvement in her re-election prospects. This applies, as far as to satisfy the demand of the supporting party is a binding condition for the head of government. Secondly, being a member of the cabinet may allow a government premium, i.e. a benefit exceeding the continuation pay off, which is available to the party if she enters negotiations in the parliament. Here, I assume that for a given continuation pay off government expenditures rise with an increase in the government premium.

In the government formation stage, proposals are submitted in a fixed order with the

\(^1\)Provided another mild condition is met.
largest party making the first proposal. If there is no government premium, parties are indifferent between having a minority or majority coalition government and randomize. Basically, my argument is that a minority government is likely to be accepted if the pay off outside the government is as high as the pay off within the government. That considerations along these line play a role in the formation of governments is supported by Strom (1990). He finds that better options outside the government, as available through institutional arrangements, make the emergence of minority governments more likely. What remains to explain is why majority coalition governments occur with higher frequency. However, in the presence of a government premium, only majority coalition governments emerge and the equilibrium offer consists in the largest party offering the smallest party to join into a majority coalition government.

Without prior information on the size of the government premium, i.e. on expected pay offs in the cabinet and in the parliament, a general neutrality result applies: On average, minority governments should not be expected to differ from majority coalition governments with respect to expenditure policies. More than this, if the head of the government is in a strong position at the government formation stage, so that a minority government is even accepted if a government premium is available, minority governments have exceptionally low expenditures.

While the neutrality result holds ex ante, I show that a rise in political instability makes the emergence of minority governments more likely. At the same time as the agents become indifferent between running a minority or majority coalition government, expenditures rise. I model political instability as a shift in electoral uncertainty with constant fragmentation, i.e.
even though the composition of the government may change, the choice is always between running a minority and a majority coalition government. In this case, one can show that once the uncertainty parameter becomes sufficiently large, the demand of the supporting party rises and she favors a termination of the government. The opposition party concedes unless the increase in the demand of the small party becomes "too large". This result prevails, no matter whether the government is replaced or it is dissolved and the elections are actually held.²

There are two features which are indispensable for the working of the model: First, seats and, therefore, the parties’ vote shares do matter for the allocation of benefits in the legislature. Here the model at hand contrasts to infinite horizon legislative bargaining models where adjustments in randomization strategies outbalance the effect of a shift in parties’ recognition probabilities, which are assumed to be proportional to representation (see Baron, 1991, Baron and Ferejohn, 1989). Secondly, this model implies that the distribution of powers in the cabinet responds to a change in electoral outlooks in a sluggish way. I assume that there is no adjustment in the composition of the cabinet such that a government premium is restored after it has been consumed by a rise in electoral uncertainty. Note, that the existence of a government premium is not explained within the model, so there are no incentives for the head of the government to retain it at a certain level. Furthermore note, that even without such an adjustment the higher demand of the supporting party, which is induced by the rise in uncertainty, is met. The government premium might be best explained as resulting from non continuities in the government’s offer function, which are caused by

²For a formal model which discusses these options see Lupia and Strom (1996) and Diermeier and Stevenson (1999, 2000).
in institutional restrictions. Therefore, treating cabinet benefits as fixed makes sense at least in certain intervals.

The basic model is set up in section II.A. Section II.B. describes how the proposal procedure on either stage affects the size of government expenditures. Section II.C. solves the bargaining game in the parliament. Section II.D. introduces negotiations in the cabinet. Using the payoffs from the budget game, the government formation game is set up in section II.E. Section III. establishes the general neutrality result and discusses how this is affected by the introduction of political uncertainty. In Section IV, we discuss the effects of a shift in re-election uncertainty. Section V. concludes.

II. Bargaining outcomes in the absence of political uncertainty

A. The model

There are 3 parties \( i = 1, 2, 3 \) in the legislature. The share in parliamentary seats is proportional to vote shares \( \pi^i \) and \( \pi^1 > \pi^2 > \pi^3 \). Furthermore, I assume that the distribution is such that no party has a majority of its own and - consequently - any two parties have a simple majority of seats. Parties are the basic actors of the model. Each party derives a utility in which enters the payoffs from the budget game, the constituency receives as a function of fiscal policies. There are no office rents\(^4\) for the party in the government. The party’s objective function is

\(^3\)The idea here is that if a party is given a certain jurisdiction she cannot be denied to claim a certain pay off, even if her threat point in the budget negotiations is not binding. Discontinuities in the offer function exist because the number of jurisdictions is discrete.

\(^4\)If office rents were to be traded against expenditure benefits when negotiating over expenditures, this model would predict that coalition governments have lower expenditures than minority governments. However, one can show that this argument does not hold if office benefits are unobservable.
quasi-linear in per head expenditures of a group specific good $g^i$, the tax payment, $\tau$ and its share of the available office rent $b^i$:

$$W^i = v(g^i) - \tau.$$ 

One may think of $g^i$ as a public good like education, police or health which has to be provided at a certain level per head of the overall population to be effective\(^5\) but which is evaluated just by a certain constituency. Government expenditures share the latter characteristic with the pork barrel.\(^6\) Utility of the constituency, $v(g^i)$ is concave in the per head outlay $g^i$, has $v(0) = 0$, $\lim_{g \to 0} v_g = \infty$, $\lim_{g \to 0} v_{gg} = -\infty$ and the utility imputation space is bounded, i.e. $\lim_{g \to \infty} W(g) = -\infty$. The budget is an expenditure-taxation policy $\{g^k, \tau\}_{k=1}^3$ satisfying budget balance, i.e.

$$\sum_k g^k = \tau.$$ 

Substituting the budget constraint into the objective function, denoting aggregate expenditures per head $G$ one can write

$$V^i = W^i(g^i, G).$$ 

It follows, that the utility of the zero budget is $V(0, 0) = 0$.

My model of government formation and budgeting consists of four main stages which are depicted in figure 1. In the first, the government is formed. In the second, a budget proposal is negotiated in the cabinet. In the third, the legislature votes on the budget bill. The budget proposal is either accepted or rejected it in which case a new bargaining round in the parliament starts. Bargaining strength in the parliament is directly proportional to

\(^5\)However, scaling in either direction does not affect any of the results of the paper.

\(^6\)See for example Shepsle and Weingast (1981).
Figure 1: Government Formation, Cabinet Negotiations, Voting on the Bill in Parliament and the Call for Early Elections
the number of seats a party has and is, therefore, directly affected by the election outcome. A majority of parties may call for early elections, in which case we enter the forth stage and acceptance of the bill is delayed until a new government is formed. With early elections, another government formation and budgeting game starts from the beginning. All moves in the game satisfy subgame perfection.

At the government formation stage, parties are selected in a fixed order to make a government proposal. The party with the highest vote share makes a proposal to any other party. If it is rejected, the party with the second highest vote share gets a chance to make a proposal. The final proposal comes from the party with the least share. If this proposal is rejected, a caretaker government is implemented. The formateur either proposes a majority or a minority government. The responding party compares the pay off offered to the continuation value, $\Lambda^j$, from rejecting and moving forward at the government formation stage with the selection of the next formateur. Every government proposal is associated with a unique expected pay off configuration from the continuation at the two stages which form the budget game.

A bill which passes the cabinet negotiation stage promises the party who has accepted the proposal a utility value $V^0$ which is smaller than the pay off for the formateur, $V^\omega$. This assumption is reasonable, as the formateur acts as the head of government and she or her party would hold to a position to influence the budget (for example, as a budget minister).

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7 I do not present a model which discusses government replacement as opposed to government dissolution like Lupia and Strom (1995), Diermeier and Stevenson (1999, 2000). Therefore, if the threat to call for new elections is credible, its consequences do not depend on the new elections actually taking place.

8 This selection process is considered by Austen-Smith and Banks (1988).
Under our assumptions on the government formation game, an equilibrium proposal provides such an unequal distribution of powers.

If the cabinet proposal is rejected in the parliament, \( j \) expects to realize \( EV_P \) in a bill which is negotiated in the parliament. If \( V^0 \) exceeds \( EV_P \), there is a premium to being a member of the coalition government. At any stage, a majority of parties may call for new elections. We denominate the expected pay off, which a party receives when early elections are called as \( \Pi^j \). A government - minority or majority government - is only stable, as long as the parties who support the government realize a pay off which exceeds \( \Pi^j \). Even though this condition is certain to hold when a government is inaugurated, there are exogenous shocks which may jeopardize the stability of the government.

We go on to sequentially solving the game at the cabinet and the parliamentary stage. Taking the results will enable us to discuss the government formation game.

B. Government expenditures

Throughout this paper we will discuss expenditure policies as resulting from the proposal of one party \( \omega \) who is selected to submit a proposal and who maximizes her own pay off subject to the restraint that the party, who receives an offer realizes a pay off which is at least as high as the value of her outside opportunity \( \hat{V} \) - i.e. her pay off from rejecting the offer. The problem for the proposal maker \( \omega \) is then

\[
\max_{\{g^\omega, g^i\}} V^\omega(g^\omega, g^i) \text{ s.t. } V^j(g^\omega, g^j) \geq \hat{V}, \text{ for } j \in C\setminus\omega
\]

The f.o.c.'s are necessary and sufficient conditions for an optimum to this problem and yield
\[ \frac{v_g(g\omega)}{v_g(g')} = \frac{\lambda}{m - 1} \]  

The shadow price be expressed as

\[ \lambda = \frac{1}{v_g(g') - 1}, \lambda \in (0, 1]. \]  

\( \lambda \) can be interpreted as the utility loss for \( \omega \) from raising the utility of the other coalition party by one unit. Stated otherwise, \( \lambda \) is the rate at which government receipts have to be devoted to the compensation of the other coalition party if \( \omega \) wishes to increase his consumption by one unit. \( \lambda = 0 \) corresponds to the case, where the responders get nothing and \( \lambda = m - 1 \) corresponds to the equal share allocation. A rise in the value of the outside option \( \hat{V} \) goes along with a rise in government expenditures if government spending is sufficiently inelastic:

**Lemma 1** If the marginal evaluation schedule for public expenditures is sufficiently inelastic, i.e. if for all \( \lambda \in (0, 1 - m) \) and \( \alpha \in (0, 1) \) it is true that \( v_g(\alpha \frac{n(k(\alpha)g\omega)}{n(g')} g\omega) > \frac{1}{\alpha} v_g(g\omega) \) where \( k(\alpha) \) is implicitly defined as \( k = \alpha \frac{n(k_g\omega)}{n(g')} \), then aggregate spending increases with a reduction in inequality.

**Proof.** See appendix A.

Intuitively, the faster marginal utility diminishes, the more expensive in terms of public outlays devoted to the consumption raising the utility of the other party is. Therefore, inelasticity of the marginal evaluation schedule is a necessary condition that utility is mainly raised via higher expenditures\(^9\) instead of lower taxes. Even though this assumption - to-

\(^9\)In fact, as shown in appendix A, consumption always rises with \( \hat{V} \).
gether with the other restrictions on \( v(g) \) - excludes a range of widely used utility functions like the logarithmic utility function, it is in accordance with the empirical observation that the price elasticity of government expenditures is low.

C. Stage 3: Negotiations in the parliament

The cabinet submits its budget bill to the parliament. The parliament either accepts or rejects the proposal. If it is accepted, the bill is implemented by the executive (i.e. each expenditure category by the responsible cabinet minister). If the cabinet bill is rejected, a new bargaining round starts in the parliament. Under the institution I describe in this section, a member of the parliament, when rejecting the government proposal knows that upon entering the parliamentary bargaining stage, he realizes an expected pay off of \( EV^{Pj} \) from entering parliamentary negotiations. For the moment, assume that a move to dissolve the parliament and call for new elections can only be made before the parties enter the new negotiations round in the parliament and has to be accepted by a majority of parties. The bargaining horizon in the parliament is short, i.e. a simple closed rule applies. Furthermore, representation in the parliament matters for expected pay offs.

For each party the probability of being in a position to make a proposal is proportional to its representation in the parliament, i.e. \( \pi^i \). If a proposal is rejected, a caretaker government is selected and each agent realizes \( V(0, 0) = 0 \).\(^{10}\) Suppose that \( i \) is the parliamentary proposal

\(^{10}\)This assumption can be replaced by the assumption that a party has to be given at least her expected pay off from calling for new elections in which case we could dismiss with our previous assumption that there is a commitment of the party not to go for new elections after considering a proposal. Note, that in this case the reservation pay off is \textit{exogenously} determined from the expected pay off at the next stage and is not determined endogenously from the equilibrium conditions of the model. As is well known, in the latter
maker. The unique equilibrium is to give enough to one other party \( j \), such that

\[
V^j(g^i, G) \geq V(0, 0), \quad j \in C \setminus \omega
\]

holds, to give nothing to the other and to maximize his own pay off subject to (4). The expected pay off from entering bargaining in the parliament is

\[
EV^{Pi} = \pi^i V_{\omega PA} + \frac{1 - \pi^i}{2} V(0, 0) - \frac{1 - \pi^i}{2} G^P.
\]

\( EV^{Pi} \) consists of the pay offs for the proposal maker, \( V_{\omega PA} \), the responder, \( V(0, 0) \), and the tax paid by the looser at the parliamentary stage, \( G^P \) where we have taken account of the fact that the proposal maker equally randomizes between both potential coalition partners. What matters for our subsequent discussion is that

\[
\frac{\partial EV^{Pi}}{\partial \pi^i} > 0
\]

for all \( \pi^i \).

D. Stage 2: Negotiations in the Cabinet

If the government is a majority coalition government, each coalition party \( i \) holds one ministry which administers the supply of the good \( g^i \) which the party prefers. We assume that the arrangement is such that the coalition party who does not head the government can realize a pay off \( V^0 \) which the party heading the government has to take as given.

case a recursion argument applied to a stationary bargaining environment would say that the equilibrium is supported by randomization strategies and has equal expected values for all parties (see Baron and Ferejohn, 1989).
For our purposes, $V^0$ can be taken as exogenous. In a budgeting game, we can give a straightforward justification in terms of decentralized expenditure proposals.\footnote{See e.g. Von Hagen, 1992.} Let the budget proposal on the floor consist of the spending proposals which the portfolio minister submit. This policy proposal corresponds to a Nash equilibrium or to its equivalent in spatial models like the lattice point (Laver and Shepsle, 1996) or an implementable policy equilibrium relative to the allocation of jurisdiction (Austen-Smith and Banks, 1990) and yields the prescribed pay off $V^0$. As these proposals give rise to inefficiencies due to a common pool problem, there is scope for a mutually beneficial budget coordination proposal. Suppose the counter proposal of the budget coordinator has to be accepted by all members of the cabinet. Obviously, his task then is to solve problem (1) with the restraint $\hat{V} = V^0$.\footnote{There is one further complication arising in a two stage budget game, where the proposal on the floor in the second round consists of proposals of all players in the first round. The player who submits the proposal in the second round could try to destroy the threat point of the other players by asking for too much in the first round. However, no such problem arises if we restrict strategies to be ex post credible if the budget is carried out under discretion. Von Hagen (1992) finds that discretion in the execution of the budget is a cause for higher expenditures. This is consistent with this model where a commitment of the head of government to overspend once his second proposal is rejected would reduce $V^0$ and, by that way, decrease overall spending.}

Although roughly descriptive of real life budgeting procedures, any model which gives rise to a government premium has to discard the possibility of a vote of confidence motion by the head of government, as it is discussed in Huber (1996). With a vote of confidence tied...
to his own proposal, the proposals of the other cabinet minister would only have declarative value and the pay off of the other ministers could be reduced up to the lower continuation value for the subsequent game in the parliament. In order to retain some value to being in the government, even in the presence of a vote of confidence, Huber himself suggests that the initial configuration of policy proposals (in this case, the related utility value) serves as a coordination point among the other parties when a vote is taken in the parliament. However, in order also to implement this policy, it would be necessary that the coalition parties can in some way commit to the coordinated policy proposal. This is at least not completely satisfactory in a game which otherwise relies on the derivation of subgame perfect equilibria. On the other hand, among the 4 states with minority governments in the sample of Roubini and Sachs, two (Italy and Sweden) have strong restrictions on a vote of confidence while in another (Norway) internal restrictions apply within the Labor Party. Although this points to a neglected variable in cross-country comparison, it strengthens this model in terms of the explanation of within country variations.

If the government is run as a minority government, the cabinet stage is unsubstancial. As the head of the government has to insure support in the parliament, he submits a budget bill which gains the support of one other party. In this case, he buys the approval of the cheapest party, which is party 3, by allocating the necessary expenditures of her preferred good. Party 3 approves, if she gets at least $EV^{P3}$ and the corresponding pay off for the head of government is $V^w(EV^{P3})$. 

17
E. Stage 1: The Government Formation Game

After elections were held the president calls the party with the highest vote share, which is party 1, to make a government proposal. Upon rejection the proposal right is passed to the second and third party, subsequently. After the proposal of the third party is rejected, a caretaker government is selected and every party realizes pay off $V(0,0)$. The government, which a proposal maker offers might be a minority or a majority government.\textsuperscript{13} If she offers a minority government, it is party 3 who will realize $EV_{P3}$ in the subsequent budget game (except for the case that party 3 herself runs the government). If she offers a majority government, the responding party knows that she receives $V^0$, unless she can claim a higher demand in the parliament. A party accepts an offer which is better than rejecting the offer. She flips a coin, if rejecting and accepting leaves her with the same pay off.

The following pay offs can be realized in the budget game:\textsuperscript{14} $\{V^\omega(V^0), V^0, -G(V^0)\}$ for the head of government, her coalition partner and the non coalition party if the government is a majority coalition government. The respective pay offs are $\{V^\omega(EV_{Pj}), EV_{Pj}, -G(EV_{Pj})\}$ with a minority government which is supported by party $j$.

1. The case of a government premium, $V^0 > EV_{P3}$.

In this case we know that $V^\omega(EV_{P3}) > V^\omega(V^0)$, so the head of government would prefer to run a minority government, and $G(EV_{P3}) < G(V^0)$, so the non coalition party prefers a mi-

\textsuperscript{13}Only some countries have a formal investiture vote. However, in this framework a party gains nothing from running a minority government unless she expects that a bill is getting support of another party.

\textsuperscript{14}The functional form is used to indicate the value $\tilde{V}$ under which the expenditure determination process (1) has assigned the corresponding pay offs.
nority government. The outcome of the bargaining game, however, is a majority government with party 1 and 3 forming a coalition:

Proposition 1. If \( V^0 > E V^P3 \), party 1 and party 3 forming a majority government is the unique equilibrium of the government formation game.

Proof. Party 3 is the cheapest party ex post, so she is always the one asked at the budgeting stage to support the budget proposal of a minority government headed by any other party than herself. Party 3, when recognized in the government formation process, offers party 2 to support a minority government if \( V^ω(V^0) < V^ω(E V^P2) \) and a majority government including any other party otherwise. Either proposal is accepted.

Party 2 does not make an offer to party 3 because the only offer party 3 would accept has to give her \( Max(V^ω(E V^P2), V^ω(V^0)) \). Party 2 asks party 1 to support a minority government at the government formation stage. This would imply that party 1 realizes the pay off \(-G(E V^P3)\) at the budgeting stage which is better than the pay off \(G(E V^P2)\) which she is certain to realize when rejecting.

Finally, party 1 offers party 3 a majority government. Offering a minority government is what party 3 also gets by rejecting so she flips a coin. Party 1 would not make a proposal which is rejected with probability of \(1/2\) because in that case she only realizes \(-G(E V^P3)\).

Therefore, the equilibrium consists of party 1 offering party 3 a majority government. ■

We may ask for the effect of other bargaining procedures. Elsewhere I derive similar results to this paper for an open horizon bargaining model with random selection.\(^{15}\) We

\(^{15}\)There I discuss the effects of a shift in the continuation value from bargaining in the parliament. The problem of analyzing a re-election motive in such an environment is that changes in representation do not
may change the bargaining power of the head of government within the present context, for example by admitting that he makes a take it or leave it offer, where upon a rejection a caretaker government is implemented and every party realizes a pay off of $V(0, 0)$. In this case,\footnote{The same result applies, if the proposal power stays with party 1 after a rejection and there is a positive probability of a negotiation break down in which case a caretaker government is established.} party one proposes a minority government which is supported by party 3. This government runs expenditures $G(EV^P_3)$ which are lower than the expenditures $G(V^0)$ which a majority government would have supported under the same circumstances. Therefore, we may state that if the head of government derives his strength from the government formation stage, he runs a minority government with exceptionally low expenditures.

2. The case of no government premium $V^0 \leq EV^P_3$.

Here, the expected value for negotiating in the cabinet for party 3 is equal to or less than her value as a legislator, i.e. $V^0 \leq EV^P_j$. One reason for such an outcome could be that it is impossible for the junior coalition partner to enforce anything in the cabinet. If 1 runs a government including 3, both are indifferent to having a minority government. Because of its indifference, 3 might reject the first offer and end up supporting a minority government headed by 2. Note, that any government which emerges runs the same expenditure policy.

**Proposition 2** If $V^0 \leq EV^P_3$, the outcome of the government formation game is either a minority government supported by 3 and headed by 1 or 2 or a majority government consisting of 1 and 3.

affect the bargaining outcome but are balanced by an adjustment in the probability that a party receives an offer (see Baron and Ferejohn, 1989 or Merlo, 1997).
Proof. Last stage: because $V^0 < EV^{P2}$, party 3 offers party 2 to vote for a majority or minority government with expenditures $G(EV^{P2})$.

Second stage: party 2 offers 1 to vote for a minority government (supported in the budget stage by 3) with expenditures $G(EV^{P3})$ and party 1 accepts because otherwise it faces tax payments of $G(EV^{P2})$.

First stage: party 1 offers 3 a minority or majority government. 3 is indifferent between accepting and rejecting the offer. If it accepts whatever offer is made the outcome is a coalition government of 1 and 3 or a minority government of 1 supported by 3 in the budgeting stage. If it rejects the outcome is a minority government 2 supported by 3 in the budgeting stage. ■

III. An Ex Ante Neutrality Result

The prediction of the model so far is that governments with a higher government premium have both higher cohesion and higher expenditures. If one fixes a reversion level $EV^{P3}$, expenditures vary directly with $V^0$ for majority coalition governments (who have $V^0 > EV^{P3}$). In the case with no government premium (i.e. $V^0 \leq EV^{P3}$), fixing $V^0$, expenditures vary with $EV^{P3}$. It is easy to see that a general neutrality result holds: If we lack information on the processes generating $EV^{P3}$ and $V^0$, it is reasonable to hold Laplace expectations on the payoffs itself. If so, there is no reason why one should expect a majority government to have higher or smaller expenditures than a minority government.

Proposition 3 Suppose that $G$ is given by $G := f(\max(EV^0, EV^{P3}))$ and let $EV^0$ and $EV^{P3}$ be equally and independently distributed on $[0,1]$. Then average expenditure for all
governments (minority and majority) is the same as average expenditure for the group of governments where randomization between majority and minority governments occurs (i.e. with \( EV^0 \leq EV^{P3} \)).

**Proof.** To save notation let \( e := EV^{P3} \) and \( v := V^0 \). \( P(e \geq v) \) gives the probability that \( e \) exceeds \( v \). Let \( f(e) \) and \( f(v) \) denote expenditure levels corresponding to \( e \) and \( v \), respectively. Conditional expenditure for the group of randomized minority and majority governments is

\[
EG(e \geq v) = \frac{\int_0^1 f(e) P(e \geq v)de}{\int_0^1 P(e \geq v)de} = \frac{\int_0^1 f(e) (\int_0^e vdv) de}{\int_0^1 e de} = 2 \int_0^1 e f(e) de
\]

and average expenditure across all governments is

\[
EG = \int_0^1 f(e) P(e \geq v)de + \int_0^1 f(v) P(v \geq e) dv = 2 \int_0^1 e f(e) de.
\]

In the simple case where \( G = Max(EV^0, EV^{P3}) \) average expenditure is 2/3 in both cases.

In spite of this neutrality result which holds in a stationary environment I go on to show how a non stationary environment affects this result. Suppose, that a majority government preexists and a shock in expectations occurs such that \( \pi^3 > \bar{\pi} > \pi^3 \) where \( \pi \) is the critical share for the smallest party above which the condition \( EV^{P3} \geq V^0 \) is fulfilled. For higher values of \( \pi^3 \) both, party 1 and 3 are indifferent between running a majority and minority government. If party 3 were successful in calling early elections, the government established after such an election would be a minority government with probability 1/2, which exceeds the ex ante probability of having a minority government, and this government would have relatively high expenditures because \( G \) rises as \( EV^{P3} \) rises above \( V^0 \). Also, if this is the case,
it should not be necessary for the elections to be actually held to increase the power of party 3 in the budget negotiations with the head of government.\textsuperscript{17} In that case, party 1 and 3 are now indifferent between running a minority and majority coalition government.

The problem with the argument, as developed so far, is that party 3 would never find support for a call for new elections. Under the institutional set up and the assumption that $\pi^2 \equiv 1/3$ party 1 and party 2 can only loose by calling for new elections. Party 2 knows that the new government would be a minority government supported by party 3 or a majority government with expenditures $G(EV^{P3}) > G(V^0)$.\textsuperscript{18} It is only in the limiting case $\pi^3 = \pi^2 = \pi^1 = 1/3$ where the fate of party 2 actually improves. But as this event happens with probability zero, party 2 never concedes into new elections. Indeed it would be puzzling, should a party who expects some other party to flourish in a minority regime agree in new elections if she knows she has to pay the price for higher expenditures.

In the next section, however, we are going to construct an example which shows that dropping the assumption $\pi^2 \equiv 1/3$ and introducing some amount of uncertainty alone might be sufficient to have both party 2 and party 3 vote against the government and call for new elections. If this is the case, 3’s threat is substantial and she can indeed claim $EV^{P3}$.

\section*{IV. The Effects of a Shift in Electoral Uncertainty}

Why then, do minority governments have such a bad reputation? In this section I develop an argument which shows that if there is an exogenous rise in electoral uncertainty this can

\begin{footnotesize}
\begin{itemize}
\item \textsuperscript{17}See Laver and Shepsle, 1996, p 210.
\item \textsuperscript{18}Unless party 2 is prepared to agree in a new election, the threat of party 3 is unsubstantial and she cannot claim more than $V^0$.
\end{itemize}
\end{footnotesize}
result in both the collapse of the present government and the emergence of a minority government with higher expenditures. Suppose, that a majority government preexists and a shock in expectations occurs such that $\pi^3' > \pi > \pi^3$ where $\pi$ is the critical share for the smallest party above which the condition $EV^{P3} \geq V^0$ is fulfilled. For higher values of $\pi^3$ both, party 1 and 3 would be indifferent between running a majority and minority government on the government formation stage. But under what circumstances would party 2 concede into new elections - what would actually be necessary to bring about any change in outcomes?\footnote{Unless party 2 is prepared to agree in a new election, the threat of party 3 is unsubstantial and she cannot claim more than $V^0$.} Under certainty and with the given institutional set up party 2 can only win if it either gets into first or third position or if it expects party 3 to reject the offer by party 1 in the case of indifference (proposition 2). On the other hand, a government of 1 supported by a stronger 3 would charge higher taxes on 2.

In the remainder of the paper I am going to construct an example which shows that introducing some amount of uncertainty alone might be sufficient to have both party 2 and party 3 vote against the government and call for new elections even if party 2 sets the probability of 3 rejecting an offer of 1 at zero.

For this we have to become more concrete. We assume that, initially, $\pi^2 = 1/3$, $\pi^3 > 1/6$ and $\pi_1 = 2/3 - \pi_3$. Furthermore, for party 3

$$EV^{P3}(\pi^3 = 1/6) < V^0 < EV^{P3}(\pi^3 = 1/3).$$

Suppose that there is uncertainty about the outcome of elections. I am going to show that if this uncertainty rises sufficiently and the demand of party 3 does not become "too..."
large”, both party 2 and party 3 vote for new elections and that the government emerging from the election has a relatively high probability of being a minority government. In deriving this result, I assume that as uncertainty rises, the fragmentation of the government remains constant, i.e. the choice is always between running a minority and a majority coalition government. In constructing the example, I do not account of those cases, where the supporting party faces a risk of loosing representation in the parliament because a quota applies. Nor do I allow that the largest party gets a simple majority. However, I feel that this assumption best suits the notion of a politically instable environment.

To keep matters simple and in accordance with the focus of the model on the presence of different interest groups I assume that members of the group always vote non strategically for the party representing their interest. Election uncertainty comes from uncertain voter mobilization.

In our example, initial vote shares are \( \pi^1 = \frac{5}{12}, \pi^2 = \frac{4}{12}, \pi^3 = \frac{3}{12} \). Let the critical value, \( \underline{\pi} = \frac{3}{12} + \epsilon \) for some small but positive \( \epsilon \). Assume that voter turn out at the last election has been \( \frac{3}{4} \) for all parties, so the vote count associated with these shares is \( \underline{x}^1 = \frac{5}{16}, \underline{x}^2 = \frac{4}{16} \) and \( \underline{x}^3 = \frac{3}{16} \), respectively. Let the expected vote count for party \( i \), \( x^i \) be a random variable with the probability distribution function \( h(x^i) = \frac{1}{(\underline{x}^i + \Delta)} \) for \( x^i \in (\underline{x}^i - \frac{1}{32} - \Delta, \underline{x}^i + \frac{1}{32} + \Delta) \). If \( \Delta \) is not too large (i.e. not exceeding \( \frac{1}{16} \)), the following cases may arise in the newly elected chamber:

\[ C_1: \pi^1' > \pi^2' > \pi^3' > \underline{\pi} \text{ with pay offs } \{V^\omega(EP^3), -G(EP^3), EP^3\} \text{ for party 1, 2 and 3.} \]

\[ C_2: \pi^1' > \pi^2' > \pi > \pi^3' \text{ with pay offs } \{V^\omega(V^0), -G(V^0), V^0\}. \]
\[ C_3: \pi^1 > \pi^3 > \pi > \pi^2 \text{ with payoffs } \{V^\omega(V^0), V^0, -G(V^0)\}. \]
\[ C_4: \pi^1 > \pi^3 > \pi^2 > \pi \text{ with payoffs } \{V^\omega(EV^{P2}), EV^{P2}, -G(EV^{P2})\}. \]
\[ C_5: \pi^2 > \pi^1 > \pi > \pi^3 \text{ with payoffs } \{-G(V^0), V^\omega(V^0), V^0\}. \]
\[ C_6: \pi^2 > \pi^3 > \pi > \pi^3 \text{ with payoffs } \{-G(EV^{P3}), V^\omega(EV^{P3}), EV^{P3}\}. \]

As we have required \( \Delta \) not to exceed \( \frac{1}{16} \), we can omit the cases where government 1 and 3 change positions. First of all, note that if \( \Delta < 0 \), the outcome is a lottery between the status quo \( C_2 \) and \( C_1 \). In that case, party 3 votes for new elections because it retains the status quo with probability \( 1/2 \) and improves with probability \( 1/2 \). As can easily be seen, the interest of party 1 and 2 is directly opposed.

It is easy to show the following

**Lemma 2** The probability of the events \( C_3, C_4, C_5 \) and \( C_6 \) is positive for \( \Delta > 0 \) and increases in \( \Delta \) whereas the probability of the events \( C_1 \) and \( C_2 \) decreases in \( \Delta \).

**Proof.** See appendix ■

Now we can evaluate payoffs for an increase in \( \Delta \). Note from the appendix that \( \Pr(C_3) < \Pr(C_5) = \Pr(C_6) < \Pr(C_4) \). For \( \Delta < 0 \), the loss for party 2 is smaller than the gain for party 3, because \( v_g(g^3) > 1 \) and, therefore, \( v_g(g_3)dG > dG.\)\(^{20} \) Now look at the gains for party 2 in the events \( C_3 - C_6 \):

\[ C_3: \text{gain}^2 = V^0 + G(V^0), \text{loss}^3 = V^0 + G(V^0) \]
\[ C_4: \text{gain}^2 = EV^{P2} + G(V^0), \text{loss}^3 = V^0 + G(EV^{P2}) \]
\[ C_5: \text{gain}^2 = V^\omega + G(V^0), \text{loss}^3 = 0 \]
\[ C_6: \text{gain}^2 = V^\omega + G(V^0), \text{loss}^3 = V^0 - EV^{P3} \]

\(^{20}\)More precisely, from (1) we have \( v_g(g^3) = 1 + \frac{1}{\lambda} \) where \( \lambda \in (0, 1) \) and from (6) we know that \( dG < dg^i \).
In $C_3$, gains and losses exactly cancel out each other. In $C_5$, party 2 gains and 3 looses nothing whereas in case 6 party 3 even wins. Remains $C_4$. It is to show that gains of party 2 exceed losses for party 3. Again, increasing the expenses for party 2 by $dG$ costs party 3 $dG$, but party 2 profits by $v_2(g^2)dg$ which is larger than $v_3(g^2)dG$. Collecting arguments, we have shown that with $\Delta < 0$, the benefit for party 3 is positive and greater than the loss for 2. By increasing $\Delta$, 2 gains more than 3 looses. So it suffices to insure with the parameters of the model, that there is some $\Delta < \frac{1}{16}$, such that $\sum_{C_i} \text{gain}^2(C_i) > G(EV^{P3}) - G(V^0)$, which is 2’s loss in $C_1$ compared to the status quo.

**Result 1** For $G(EV^{P3}) - G(V^0)$ sufficiently small, there is an increase in $\Delta$ such that party 3 and party 2 vote for a termination of the government.

So it does not even take an increase in expected pay offs for party 3 to elicit a government crisis. On the other hand, there is always an increase in $\Delta$, such that party 2 accepts an increase in $\pi^3$ and still votes for new elections.

Taking one further look at the events $C_3 - C_6$, we note that randomization between majority and minority government occurs in $C_4$ and $C_6$. These are also the events, in which expenditures rise. Thus, political instability as measured by re-election uncertainty $\Delta$, increases the likelihood of government break ups with increased probabilities of minority governments emerging and (generally) with the promise of increased expenditures. We subsume this result in the final proposition:

**Proposition 4** If result 1 applies, a rise in re-election uncertainty with unaltered fragmentation increases the conditional probability that a minority government forms. Furthermore, if the conditions for a emergence of a minority government are met, expenditures increase.
Proof. See discussion above.

It is also obvious from our discussion in the previous section, that this is not true of minority governments in general and is wrong for minority governments, whose emergence is founded in the relative strength of the head of government. It might be, however, that political uncertainty and emergence of minority governments play together in a subtle way which would account for those unsystematic observations of the performance of minority governments which gave them their bad reputation.

V. Conclusion

In this paper I have presented a comprehensive approach, which systematically relates the form of government to the expected pay offs from budget negotiations. I show that in the absence of re-election uncertainty, there are no systematic differences in spending between minority and majority coalition governments. A sufficient increase in re-election uncertainty makes minority governments more attractive and, as the government premium is consumed, raises spending. The prediction is, therefore, that minority governments, whose emergence is related to political instability would show above average spending. On the other hand, minority governments may be an indicator of political stability and, in that case, have below average expenditures. This suggests, that the empirical literature on the political determinants of government expenditures is mislead. Its focus on the presence of minority governments as an indicator of political instability seems to reverse the causality.
VI. Appendix

A. Proof of lemma 1

All allocations which are proposed in equilibrium are a solution to (1). Now consider the expression for aggregate expenditures as a function of $\lambda$:

$$G(\lambda) := g^\omega + (m - 1)g^j = v_g^{-1}((1 + \lambda)) + (m - 1)v_g^{-1}((m - 1 + \frac{m - 1}{\lambda})).$$

Using $v^{-1}(a\lambda) = \frac{a}{v_g}$ one gets

$$\frac{\partial G}{\partial \lambda} = \frac{1}{v_g(g^\omega)} - \frac{(m - 1)^2}{\lambda^2} \frac{1}{v_g(g^j)}$$

(6)

Noting that the f.o.c.’s imply that $\frac{\lambda}{m - 1} = \frac{v_g(g^\omega)}{v_g(g^j)}$ and using the elasticity of marginal utility of consumption $g$, $\eta(g) = -\frac{v_g(g^j)}{v(g)}\frac{1}{\eta(g^\omega)}\frac{v_g(g^\omega)}{v(g^\omega)}$ one finds that $\frac{\partial G}{\partial \lambda} > 0$ if

$$\frac{g^j/\eta(g^j)}{g^\omega/\eta(g^\omega)} > \frac{\lambda}{m - 1}$$

(7)

(7) implies the statement in the lemma: For $(g^\omega, g^j)$ with $g^j = \frac{\alpha}{\eta(g^\omega)}g^\omega$ where $\alpha = \frac{\lambda}{m - 1}$ the f.o.c. (2) are fulfilled but condition (7) would only hold as an equality. Consequently, $(g^\omega, g^j)$ simultaneously satisfy the f.o.c.’s and (7), if at $(g^\omega, \alpha\frac{\eta(g^j)}{\eta(g^\omega)}g^\omega)$ we have $v_g(\alpha\frac{\eta(g^j)}{\eta(g^\omega)}g^\omega) > \frac{1}{\alpha}v_g(g^\omega)$.

B. Proof of Lemma 2.

$$\Pr(C_3) = \Pr(\frac{\delta x_1 - \delta x_1}{1 + \delta T} > -\frac{1}{16}) \cap \Pr(\frac{\delta x_2 - \delta x_2}{1 + \delta T} > \frac{1}{8}) \cap \Pr(\frac{\delta x_3}{1 + \delta T} < -\frac{1}{16})$$

where $\delta T = \delta x_1 + \delta x_2 + \delta x_3$ and $\delta x^i = x^i - x^i$. Ignoring $\delta T$, the separate probabilities are given by

$$\Pr(\delta x_1 - \delta x_3 > -\frac{1}{16}) = 1, \Pr(\delta x_3 - \delta x_2 > \frac{1}{16}) = 2(\frac{\Delta}{\delta T + \Delta})^2$$

and
\[
\Pr(\delta x_2 < -\frac{1}{16}) = \frac{\Delta}{\frac{1}{16} + \Delta}. \text{ The conditional probability of } \Pr(\delta x_3 - \delta x_2 > \frac{1}{6} | \delta x_2 < -\frac{1}{16}) = \frac{\Delta}{\frac{1}{16} + \Delta}, \text{ so } \Pr(C_3) = \left(\frac{\Delta}{\frac{1}{16} + \Delta}\right)^2.
\]

For the event \( C_4 \) we have
\[
\Pr(C_4) = \Pr(\frac{\delta x_1 - \delta x_3}{1 + \delta T} > -\frac{1}{16}) \cap \Pr(\frac{\delta x_2 - \delta x_3}{1 + \delta T} > \frac{1}{6}) \cap \Pr(\frac{\delta x_2}{1 + \delta T} > -\frac{1}{16})
\]

Again, \( \Pr(\delta x_1 - \delta x_3 > -\frac{1}{16}) = 1 \), the conditional probability \( \Pr(\delta x_3 - \delta x_2 > \frac{1}{16} | \delta x_2 > -\frac{1}{16}) = \frac{1}{2} \frac{\Delta}{\frac{1}{16} + \Delta} \) and
\[
\Pr(\delta x_2 > -\frac{1}{16}) = \frac{1}{2} \frac{\Delta}{\frac{1}{16} + \Delta}. \text{ Thus, } \Pr(C_4) = \frac{1}{2} \frac{\Delta}{\frac{1}{16} + \Delta}^2.
\]

For the event \( C_5 \) we have \( \Pr(C_5) = \Pr(\frac{\delta x_2 - \delta x_3}{1 + \delta T} > \frac{1}{16}) \cap \Pr(\frac{\delta x_1 - \delta x_3}{1 + \delta T} > -\frac{1}{6}) \cap \Pr(\frac{\delta x_2}{1 + \delta T} < 0) \) and for the event \( C_6 \) we have, quite similarly, \( \Pr(C_6) = \Pr(\frac{\delta x_2 - \delta x_3}{1 + \delta T} > \frac{1}{16}) \cap \Pr(\frac{\delta x_1 - \delta x_3}{1 + \delta T} > -\frac{1}{6}) \cap \Pr(\frac{\delta x_2}{1 + \delta T} > 0) \) where the probability \( \Pr(\delta x_3 - \delta x_2 > -\frac{1}{6}) = 1 \) and \( \Pr(\delta x_2 - \delta x_1 > \frac{1}{16}) = 2 \left(\frac{\Delta}{\frac{1}{16} + \Delta}\right)^2 \). So \( \Pr(C_6) = \Pr(C_5) = \left(\frac{\Delta}{\frac{1}{16} + \Delta}\right)^2 \).

Comparison immediately shows that \( \Pr(C_3) < \Pr(C_4) = \Pr(C_6) < \Pr(C_4) \).

Furthermore, \( \Pr(C_1) = \frac{1}{2} - \Pr(C_6) \cup \Pr(C_3) \cup \Pr(C_4) \) and \( \Pr(C_2) = \frac{1}{2} - \Pr(C_5) \).

**References**


