Secession and Value

Gerald Pech*

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Abstract

A model with random proposals, where secession of coalitions is the only threat, yields exploitative allocations. If proposals for coalition S involve randomization, the responders' pay off vector consists of Shapley values for coalition sizes ranging from 1 to $|S| - 1$.

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*Department of Economics, NUI Galway, Galway, Ireland, tel: 00353-91-557716, 00353-91-524130, e-mail: gpech@nuigalway.ie. Part of this research was financed by the Graduate College Bochum and Dortmund and by the May Wong-Smith fund, University of St Andrews. I wish to thank Matthias Brückner, Jim Jin, Laurence Lasselle, Christoph Lülfesmann, Ashley Piggins and Thomas Romer for comments which I received on this and earlier versions of this paper.

1 Introduction

The right to secede is often seen as a powerful device to prevent exploitation of a minority by the majority (Buchanan/Faith, 1987). We show, however, that in a world where societies are allowed to redistribute income with no other safeguard than the right to secede, resulting distributions may well be highly inequitable and, perhaps more surprisingly, exploitative. Consider, say, a state's right to secede from the Union. If its citizens feel that as a community they pay more for services from the Union than they would if they provide for those services themselves there is the temptation for them to unanimously opt for secession.[1] They should, however, realize that once the secession has taken place, they will be able to claim within the secession only what they can defend by employing the threat of further secession from the secession, and so forth.[2] We characterize stable cost allocations in a game like this. The state will stay in the Union even if its citizens end up in a situation where within the Union they pay more for the provision of services than if a secession were actually to take place.

[1] To simplify we ignore an impact of jurisdiction size on private income.

[2] Buchanan/Faith raise the issue of exploitation within a secession in a footnote but they do not consider differentiation between potential seceders. Atkinson (1995) shows that if the seceders are constrained in the choice of the tax system, redistribution is sustainable in the original society.

We pursue our argument in a cost allocation framework where a coalition provides a (local) public good for its members. A proposal maker i is randomly selected to propose tax payments for coalition S. The only objection against the proposal is for a subcoalition of $R \subseteq S \backslash \{i\}$ players to secede. However, while players can commit to a secession, they cannot commit to any particular, new cost allocation. Instead they know that a tax scheme for the seceding coalition R will be proposed by a randomly selected proposal maker who proposes under the threat of further secession. We show that the equilibrium tax scheme is exploitative, i.e. any R pays more than its stand alone cost. We derive conditions under which the equilibrium proposal involves random differentiation across agents as opposed to discrimination against certain agents. If the proposal maker assigns pay offs randomly, the resulting cost allocation for the responders corresponds to the stage wise calculated Shapley value for coalition sizes running from 1 to $|S| - 1$.

It is worthwhile highlighting the difference between the secession approach and the standard bargaining approach. In the bargaining game with random proposal makers by Hart/Mas-Colell (1996), if each responder can trigger formation of the coalition $S \backslash \{i\}$ by rejecting i's proposal, agents realize their Shapley value in the "subgame" restricted to $S \backslash \{i\}$. There, coalition $S \backslash \{i\}$

can realize its stand-alone value. If, as in our case, collective action is needed which requires all agents in a coalition to partake in a secession - the power of any individual agent is reduced and the proposal maker may "buy off" some agents while making the whole coalition pay more than their stand-alone cost. Considering secession as a bargaining move is particularly appealing as it is the most natural threat of a coalition in an institution-free setting.

Our framework reverses the order of moves in Maskin (2004): In our case, players are free to leave while in Maskin they enter in an arbitrary order. The solution concept is an application of binding agreements (Ray/Vohra, 1997) to a setting where (sub)-coalitions act under the constraint that they cannot pre-commit to a final allocation. The following example illustrates our ideas:

Example 1 *Suppose that* $N = \{1, 2, 3\}$, $C(1) = C(2) = C(3) = 6\$$, $C(1, 2) = C(1, 3) = C(2, 3) = 7\$$ *and* $C(1, 2, 3) = 11\$$.

In this example, C is subadditive but not concave and the core is empty[3]. Let 3 be the proposal maker for the grand coalition. If 1 pays 6\$ this leaves her indifferent towards seceding as a singleton. Now consider the position of 2. If 2 secedes with 1, they pay 7\$ together. So if they can sign a contract

[3] In order to be in the core, individual payments x_1, x_2, x_3 would have to satisfy $x_1+x_2 \leq 7\$$, $x_1 + x_3 \leq 7\$, x_2 + x_3 \leq 7\$$. Summing up these conditions gives $2(x_1 + x_2 + x_3) \leq 21\$$ violating the break even requirement $x_1 + x_2 + x_3 = 11\$$ (see Young 1994).

they can allocate 7\$ between them, and given that 1's cost of staying is 6\$, 2 may pay no more than 1\$ in a secession. On the other hand, suppose that they cannot sign a contract but have to agree on a secession knowing that nothing else but the threat of further secession will protect them from exploitation in the society $\{1, 2\}$. After a secession, 1 and 2 have an equal chance of imposing an allocation in their society. That is, if 1 imposes, the cost allocation is $(x_1 = 1\$, x_2 = 6\$)$ and if 2 imposes, the cost allocation is $(x_1 = 6\$, x_2 = 1\$)$. Thus the expected payment of 2 in a secession with 1 is 3.5\$. So if 2 pays no more than 3.5\$ in N, she is dissuaded from seceding with 1 (note that 1 would still prefer to secede with 2, but is stuck on her own). With this arrangement, 1 and 2 together pay 9.5\$ which is more than their stand alone cost of 7\$ and 3 would only have to pay 1.5\$ herself.

2 The model

N is the set of players. The cost function $C(S)$ for $S \subseteq N$ is subadditive, i.e. $C(S \cup T) \leq C(S) + C(T)$ for $S \cap T = \emptyset$. We assume that agents are risk neutral and choose among options so as to minimize their expected payment.

Definition 2 ***Continuation pay off*** *for agent $i \in S$ from seceding with* *coalition S is $E_S x^i$. Let $s = |S|$. For $s = 1, E_{\{i\}} x^i = C(i)$. For $s > 1$, $E_S x^i = p^i a^i_{S,i} + \sum_{j \in S \setminus i} p^j a^i_{S,j}$ where $a^i_{S,j}$ is the expected payment for i if j is proposer for S.*

We assume that for every $j \in S$, $p^j = \frac{1}{s}$. A secession $R \subset S$ forms if in a proposal for S every member of R gets less than their continuation pay off with R. Formally, we define for player j:

Definition 3 ***Maximum willingness to pay*** *δ^j of $j \in S$ conditional on a proposal for S, vector $\mathbf{a}_S$, is*

$$\delta^j(\mathbf{a}_S) = \max_{R \subset S} \left[E_R x^j | \delta^k(\mathbf{a}_S) < a^k_S \text{ for all } k \in R \right].$$

Willingness to pay within S is conditional on the proposal $\mathbf{a}_S$ itself as only to the extent that agent j finds player to secede with her in coalition R can she raise coalition R as a threat and limit her payment to her continuation pay off in R. A player might be included in a threat by more than one coalition. Entry into a secession is free.

Definition 4 ***Equilibrium proposal*** *for S by i. Secession in $R \subseteq S$ is blocked under $\mathbf{a}_S$ if for all $Q \subseteq R$, $j \in Q$, $a^j_S \leq \delta^j(\mathbf{a}_S)$. A proposal maker i selects a proposal $\mathbf{a}_S$ to maximize over all $R \subseteq S$ her pay off $\Pi^i | R$ where*

$\Pi^i|R = \max_{a_S} \sum_{j \in S \setminus i} a_S^j - C(R)$ *for which* $\mathbf{a}_S$ *blocks secession in* R. i *randomizes with equal probability between proposals if she is indifferent between alternative payment assignments.*

3 Results

What are the incentives for a proposal maker to prevent secession?[4]

Lemma 5 *With subadditivity, every player is assigned a payment such that she is prevented from seceding.*

Proof. Suppose that under an offer $\mathbf{a}_N$, coalition $S \subset N$ wants to secede. If S's secession is prevented, members are prepared to pay at least $\sum_{j \in S} a^{j\prime} \geq C(S)$. For $s > 1$ this relationship is strict: one can always charge one agent more than her $E_S x^j$ without causing S to secede. If S secedes this does not increase the willingness to pay of any player in $N \setminus S$. But under subadditivity the additional cost of serving S is $C(N) - C(N \setminus S) \leq C(S)$. ■

By lemma 5, the proposal maker i wants to prevent secession. Therefore, she has to prevent the formation of a coalition T of $n-1$ players. Say $n-2$ players in T pay more than $E_T x^j$, $j \in T$, so by definition 3 their

[4] We break cases of indifference assuming that an inclusive proposal is being made.

willingness to pay is $\delta^j(T) = E_T x^j$ if the remaining player k pays more than $E_T x^k$. Therefore, k must pay no more than $E_T x^k$. The argument holds for decreasing coalition sizes, $|T| = n-2, ..., 1$. Let $\boldsymbol{\pi}_{N\backslash\{i\}}$ be a ranking with player $j \to \pi(j)$, $\pi(j) \leq n-1$ and coalitions $B^{\pi(j)} = \{\pi = 1, .., \pi(j)\}$ such that j pays $a_N^j = E_{B^{\pi(j)}} x^j$, $j \in N\backslash\{i\}$. Clearly, this payment scheme, induced by $\pi_{N\backslash\{i\}}$, blocks secession in N and each player is pivotal for the breakaway of one coalition. We go on to give conditions under which any equilibrium payment proposal is so described by an arbitrary ranking of players. That is, players are payed according to their prospect in coalitions of decreasing size, where they are pivotal, and the proposal maker is indifferent towards assigning any player the pay off corresponding to a particular coalition size. This is the case if C is additive in an individual and a symmetric collective cost term displaying increasing returns to co-operation:

Proposition 6 *Let the cost function be* $C(S) = \sum_{j \in S} c^j - f(s)$ *with* f *convex (type I). The proposal maker assigns pay offs as follows: For an arbitrary ranking of players, each player is assigned her Shapley value in the coalition comprised of herself and all lower ranked agents.*

Proof. A ranking $\boldsymbol{\pi}^*_{N\backslash\{i\}} \in \arg\max\limits_{\boldsymbol{\pi}_{N\backslash\{i\}}} \sum_{\pi(j)=1}^{\pi(j)=n-1} E_{B^{\pi(j)}} x^j$ and the pivotal payment scheme it induces is a candidate for an equilibrium. Assume that

$E_T x^j \leq E_S x^j$ for $S \subset T, j \in S$ (condition 1). Then no deviation pays where j is charged $a^j = E_T x^j$ with $T \supset B^{\pi(j)}$. Next, suppose $a_N^k = E_S x^k$ and $a_N^l = E_T x^l$ with $|S| < |T|$ but that $S \nsubseteq S \cap T$, i.e. either there is $j \neq l$, $j \in S, j \notin T$ or $j, l \in T \backslash S$ and $k \in S \backslash T$. Suppose that secession in N is blocked which rules out the latter case. In the former case, secession is still blocked for $a_N^k = E_{S \backslash \{j\}} x^k$ and Π^i increases. So $\boldsymbol{\pi}^*_{N \backslash \{i\}}$ induces an equilibrium payment scheme.

Setting $t = \pi(t)$ to simplify notation we can now construct $E_{B^t} x^t$ for increasing coalition sizes. Suppose that for $s < t$, ranking is random and condition 1 holds. Then $E_{B^t} x^t = \frac{1}{t} \left(g(t) + c^t \right) + \frac{1}{t} \sum_{k=1}^{k=t-1} \left(g(t-k) + c^t \right)$ where $g(t) = -f(t) + \sum_{s<t} (c^s - E_{B^s} x^s) = -f(t) - \sum_{k=1}^{k=t-1} g(t-k)$ is the pay off (excluding c^t) for t as a proposer in B^t. It is immediate, that $E_{B^t} x^t = c^t - \frac{1}{t} f(t)$. So player t's ranking does not affect overall charges and condition 1 holds for convex f. For $n = 3$, the proposal maker randomizes. This completes the induction. Finally, applying the recursive definition of the Shapley value (Sprument 1990), $\phi^j(T) = \frac{1}{t} \left[C(T) - C(T \backslash j) + \sum_{S \subset T, j \in S, s=t-1} \phi^j(S) \right]$ gives $\phi^t(B^t) = c^t - \frac{1}{t} f(t)$. ■

The cost function of type I with limiting cases $c = 0$ and $f = 0$ is the only type for which the proposal maker randomizes over rankings:

Proposition 7 *For $n > 3$ every equilibrium proposal is a pivotal payment scheme with random rankings only if the cost function is of type I.*

Proof. It is a necessary condition for the proposal maker to randomize that for all permutations of $\boldsymbol{\pi}_{N\backslash\{i\}}$, $\sum_{\pi(j)=1}^{\pi(j)=n-1} E_{B^{\pi(j)}} x^j = const.$. We can use the fact that if all proposals are inclusive (by lemma 5) $\sum_{k \in S} E_S x^k = C(S)$, for establishing the following conditions: For $n = 3$, the proposal maker always randomizes. For $n = 4$ and player 4 proposing the condition for randomization is $C(1,3) + C(2) = C(1,2) + C(3) = C(2,3) + C(1)$ which implies that $C(1,2,3)$ is of type I. For $n = 5$, the condition given for $S = \{1,2\}$ has to hold for each $S \subset N\backslash\{i\}, |S| = 2$. Let $k, j \notin S$ so the condition is $3C(j \cup S) + C(k) + \frac{1}{3}[C(1,2,k) + \sum_{l \in S} C(k \cup S\backslash l)] = 3C(k \cup S) + 2C(j) + \frac{1}{3}[C(1,2,j) + \sum_{l \in S} C(j \cup S\backslash l)]$ for any $j, k \in N\backslash\{i\}$ and that C restricted to subcoalitions of $N\backslash\{i\}$ is of type I. This implies that $C(N\backslash i)$ is of type I. For $n > 5$: Let Ω^t be the set of all cost functions C which for t players satisfy the conditions for randomization. Suppose that for $|N| - 2$ it has been established that the cost function is of type I, i.e. that all $C(T) \in \Omega^t$ are of type I. As the randomization condition involves comparisons between rankings interchanging two agents, any such cost function must satisfy $(n-1)(n-2)/2$ constraints of the form (for any

$j, k \in N\backslash\{i\}$ with $S^{jk} = N\backslash\{i, j, k\}$): $\alpha\left(K(S^{jk} \cup j) + \sum_{t \in S \cup j} c^t\right) + \Xi(j) = \alpha\left(K(S^{jk} \cup k) + \sum_{t \in S \cup k} c^t\right) + \Xi(k)$ where $\Xi(j)$ and $\Xi(k)$ are terms containing the cost of subcoalitions up to size $n-3$. By proposition 6, there is C of type I, which fulfills these conditions, i.e. $C \in \Omega^{n-1}$. Now suppose there is a cost function C' not of type I for which $\alpha\left(K(S^{jk} \cup j) + \sum_{t \in S} c^t + \sigma(S^{jk}) + \gamma^j\right) + \Xi'(j) = \alpha\left(K(S^{jk} \cup k) + \sum_{t \in S} c^t + \sigma(S^{jk}) + \gamma^k\right) + \Xi'(k)$. First, C' when restricted to a subcoalition of $N\backslash\{i\}$ is of type I, so we can pick a cost function such that $\Xi'(k) = \Xi(k)$ and $\Xi'(j) = \Xi(j)$. But then it must be that $\sigma(S^{jk}) + \gamma^j = \sigma(S^{jk}) + \gamma^k = const$ which shows that C' has a symmetrical part $K' = K + \sigma + \gamma$, so C' is also of type I. Finally, if $C(N\backslash i)$ is of type I, in order to hold for all $i \in N$, $C(N\ )$ is as well. ■

Although under the condition given the proposal maker selects the ranking for assigning pay offs randomly, any particular pay off structure is not only exploitative but also inequitable. If not only the proposal maker randomizes, but she herself is randomly selected before the game (C, N) starts, the ex ante value for each player of playing the extended game is her Shapley value in (C, N). This coincides with the borderline case of no externalities in Maskin (2003, Theorem 3). But as randomization is endogenous here, the Shapley value obtains only under strong conditions.

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